Profiles —

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Creator of Modern Applied Mathematics

On the Centenary of the Birth of Academician A. N. Tikhonov



Andrei Nikolaevich Tikhonov.

The multifaceted activities of the outstanding Soviet mathematician, Academician Andrei Nikolaevich Tikhonov were focused not only on fundamental mathematical problems but also on strategic issues facing national science. The workers of the Keldysh Institute of Applied Mathematics, RAS, to which he dedicated forty years of his life, and Moscow State University (MSU), where he worked for more than seventy years, have the utmost respect for this remarkable man, whom they recall with warmth and gratitude. Many of them worked under Tikhonov for dozens of years. A collection of Tikhonov's works [1] and a book of recollections about him [2] were published. However, his memory lives of primarily in the scientific schools and mathematical disciplines that he created and in the system of mathematical education. He defined the prospects for the development of this system in secondary and higher school for decades to come. This is exactly what we would like to recall in looking back at the life of this brilliant mathematician, talented leader, and ardent

Some episodes from Tikhonov's biography. Tikhonov was born on October 6, 1906, in the town of Gzhatsk, Smolensk province, into the family of an unsuccessful merchant. When he was 15, he entered the

Mathematics Department of the MSU Faculty of Physics and Mathematics. His scientific work began in his second year of studies, when he became a member of a seminar on topology under the guidance of P.S. Aleksandrov, a future academician and topologist known worldwide. In 1925, Tikhonov obtained his first scientific result. His findings were published in *Mathematischen Annalen*, and soon F. Hausdorf included them in his classic textbook on the theory of sets. Tikhonov's discovery concerned some of the structural properties of topological spaces. Since then Tikhonov's topology and Tikhonov's cube are integral parts of general topology.

In 1937, Tikhonov was appointed professor and head of the Department of Mathematics of the MSU Faculty of Physics. That same year, the Institute of Theoretical Geophysics of the USSR Academy of Sciences was established on the initiative of Academician O.Yu. Shmidt, the well-known explorer of the Arctic and founder of the Moscow Algebraic School. On his invitation, Tikhonov worked in this Institute and soon took charge of the branch of mathematical geophysics. In 1939, he was elected a corresponding member of the USSR Academy of Sciences, in the Branch of Mathematical and Natural Sciences (the sector of geological and geographical sciences).

In 1948, Tikhonov became involved in the atomic project. In 1953, he was awarded the Stalin Prize of first degree, the title of Hero of Socialist Labor, and the Order of Lenin for "exceptional merit in fulfilling a special assignment of the Government."

In the same year, the Division of Applied Mathematics (DAM) at the Steklov Mathematical Institute (MIAN) was set up to address strategic issues related to applied mathematics. Academician M.V. Keldysh was appointed director of the new division. Between 1953 and 1963, Tikhonov was the DAM deputy director for research. In 1966, the division was upgraded to the Institute of Applied Mathematics (IAM) of the USSR Academy of Sciences. Between 1978 and 1989, Tikhonov held the post of IAM director. In 1966, he was awarded the Lenin prize for a series of works on ill-conditioned problems and was elected a full member of the USSR Academy of Sciences.



The Tikhonov family, 1910. Parents: Nikolai Vasil'evich and Mariya Nikolaevna; children: Nikolai (left) and Andrei (right)

Andrei Tikhonov at 18.

In 1970, Tikhonov initiated the Faculty of Computational Mathematics and Cybernetics at Moscow State University and was its dean from 1970 through 1990.

In 1986, he was awarded his second Star of Hero of Socialist Labor and the Order of Lenin for "exceptional merit in the development of mathematical science and training of scientific workers." Many other high state awards marked Tikhonov's scientific career: six Orders of Lenin, the Order of the October Revolution, and three Orders of the Red Banner of Labor.

Until his last days, he worked at the Institute of Applied Mathematics.

Tikhonov died on October 7, 1993, and was buried in Novodevich'e Cemetery.

Difference scheme theory. In the 20th century, science in general and applied mathematics in particular started to play an unprecedented role. One can say without exaggeration that the destinies of the world became dependent on the advances of science.

In 1989, when the fortieth anniversary of the first Soviet nuclear device test was commemorated, it was disclosed that more than 500 000 people, among them 8000 scientists, were involved in this project. Tikhonov was one of them.

The enormous power of this weapon became evident after the atomic bombardment of Hiroshima and Nagasaki. The sovereignty and the very existence of our country depended on our possessing these arms. I.V. Kurchatov took charge of the intense scientific research conducted in the framework of the Soviet nuclear project.

In 1947, the design of the Soviet atomic bomb was approaching conclusion. In 1948, a seminar was held at Kurchatov's design office to discuss, among other things, the power of the nuclear explosion. The participants examined the model developed in the theoretical division of the Institute of Physical Problems under the supervision of Academician L.D. Landau. In fact, the model was a system of nonlinear equations that had no analytical solution. Tikhonov, who also took part in the seminar, proposed to make a direct numerical calculation of the system of equations in partial derivatives using the finite difference method in Lagrangian coordinates. Landau was quick to respond to this idea, calling it a scientific breakthrough. Landau suggested that Tikhonov himself tackle the problem, and Tikhonov accepted the proposal.

In June 1948, special laboratory no. 8, attached to the Complex Geophysical Expedition of the Geophysical Institute of the USSR Academy of Sciences, was set up following the resolution of the USSR Council of Ministers.

Tikhonov, who at that time was a corresponding member of the Academy of Sciences, was appointed head of the new laboratory. Today, it is hard to imagine the colossal difficulties that the scientists had to overcome. Computers did not exist, and all calculations were made by calculating teams using Mercedes electromechanical computing machines. This meant very stringent requirements on the calculation techniques used. These techniques had to be economical, so that the volume of calculations could be within the team's capability. They had to be stable to prevent calculation errors and secure correct solutions. Finally, discrete objects, with which the calculating machine or the computer are working, and smooth differentiable functions—the language in which the laws of Nature are described—belong to different mathematical worlds. Naturally, the question arises: what properties of the continuous world must the discrete model primarily reflect? All these can be considered eternal problems of applied mathematics. Their solution depends on the precision level required by praxis, on the calculating tools available to researchers, and on the level of science.

Tikhonov and his talented student A.A. Samarskii managed to find a very felicitous solution—homogeneous implicit conservative schemes. Homogeneity, that is, the possibility to make calculations in every point of a difference grid using one algorithm, made it

possible to find the solution easily and uniformly. Implicitness secured stability, the possibility to use big grid steps, and, consequently, acceptable volumes of calculations. Conservativeness dealt with the need to obey the conservation laws governing the initial continuous equations in a discrete model. In other words, the discrete model must transmit "the physics of the process."

The idea of conservativeness had a fortunate destiny. Academician Samarskii and his disciple Yu.P. Popov (now, a RAS corresponding member) introduced the concept of complete conservativeness and constructed corresponding difference schemes making it possible to not only transmit analogues of conservation laws (for example, mass, impulse, and full energy) but also a number of additional ratios imposed by physical laws. The theory of conservative difference schemes is still being actively developed [3].

We can only wonder how rapidly and successfully the important state task was fulfilled. Tikhonov and Samarskii developed the methods of calculation of equation systems, and V.Ya. Gol'din, O.P. Kramer, and N.N. Yanenko provided support for these calculations. Beginning from scratch in 1948, the researchers performed the initial calculations of explosion equations for the plutonium ball as early as 1949 and, some time later, for the fission–fusion–fission "product." In 1953, this work continued at the Applied Mathematics Division of the Mathematical Institute. Several of Tikhonov and Samarskii's works on the theory of difference schemes became classics and were included into textbooks. Speaking about the causes of the great successes achieved in so short a period, Tikhonov used to stress the fundamental role of the our scientists' "physical approach" to these kinds of problems, as distinct from the "mathematical approach" followed by their American colleagues.

The deep understanding of the nature of physical processes enabled Tikhonov's team to construct sensible models of physical phenomena and efficient algorithms for their investigation. For many years the IAM researchers, jointly with the teams of Academicians N.G. Basov and E.P. Velikhov and with certain other leading research centers, worked on controlled thermonuclear fusion. This work led to the creation of a new research technology—computational experiment. On the one hand, the latter has the advantages of theoretical analysis, since a scientist knows what equations are used and how they are calculated. On the other hand, the computational experiment comes close to a full-scale experiment and sometimes even outstrips it in the volume and completeness of information.

One vivid example of the brilliant solution of a physical problem was the discovery of a new physical phenomenon in the course of a computational experiment: the formation of a self-sustained electrically con-



Academician A.A. Samarskii, RAS Corresponding Member Yu.P. Popov, and Nobel Prizewinner Academician N.G. Basov congratulate Academician A.N. Tikhonov on his 75th anniversary, 1981.

ductive stratum (T-stratum) during the movement of a compressible medium in magnetic field. This discovery, made by the team under Tikhonov and Samarskii, was registered no. 55 in the USSR State Register of Discoveries in 1968.

Tikhonov paid much attention to the creation of the "scientific medium" and to raising the level of theoretical research in applied mathematics in our country. He suggested that *Journal on Computational Mathematics and Mathematical Physics* be launched, and so this journal appeared in 1961; nowadays, it is almost impossible to imagine how these two major spheres of science could progress without this periodical.

Years passed, bringing about new problems. Scientists received new instruments—clusters, multiprocessor complexes, and software—that can use grids with dozens of millions of nodes. However, the ideas and traditions established by Tikhonov in this field of science are very much alive and are actively being developed at the Institute of Applied Mathematics and the Institute of Mathematical Modeling, which was created by Samarskii in 1993, and in other research centers in Russia and abroad. In many cases, history repeats itself: the potential of the country and its prospects for development once again largely depend on the status of applied mathematics and computer science.

Ill-conditioned problems. In the world of mathematics, the name of Tikhonov is first of all associated with the theory of ill-conditioned problems. In fact, Plato had already been aware of the role of so-called inverted problems in obtaining new knowledge. He proposed a kind of mental experiment, the myth of a cave. People chained inside the cave can only see the shadows of people that pass by the entrance or the shadows of objects that they carry. Question: how adequate are the ideas of the inmates of the outside world? This myth

metaphorically reflects a typical situation emerging in the course of interpreting experimental results. Imperfect tools and sensory organs place the experimenters in the position of the cave inmates.

Tikhonov felt interested in such problems in the 1940s, when he worked for the Meteorological Service and, later, at the Institute of Geography. His first studies in this area concerned the history of climate and permafrost. At that time, active debates were held on the origins of permafrost and its relation with the previous climatic coolings. The task was posed to identify the Earth historic climate judging by the current temperature/depth gradient. Tikhonov's work in this area produced classic results; in particular, he found the conditions under which there is only one solution to the task of the temperature regime restoration.

Early in the 20th century, mathematics adopted the concept of correctness of problems of mathematical physics proposed by the French mathematician J. Hadamard. In accordance with it, a problem was considered correct if it had a solution and if this solution was the only one and stable in relation to the parameters of the problem. Incorrect problems were considered mathematical curiosities having nothing to do with applications. It turned out, however, that many problems of applied mathematics, first of all, those related to the analysis of experiment results, were not correct according to Hadamard.

In the early 1960s, Tikhonov turned back to inverse problems, with which he had dealt in the 1930s and 1940s. He was able to change radically the very approach to such problems. Before him, mathematicians attempted to find correct solutions to problems with the ill-conditioned right part of the equation. Tikhonov thought it necessary to account for the accuracy of data assignment. If the source data were approximated, the operator describing the process had to be replaced by an approximated one so the problem could become correct. This transformation procedure became known as the "Tikhonov regularization method." The regularization method is applied to a very wide range of disciplines: geophysics, tomography, astrophysics, economics, and optimal control. The regularization method is superbly efficient in resolving the classic problem of linear algebra: the solution of ill-conditioned systems of linear equations.

The theories proposed by Tikhonov have been rapidly developing. The most important point here is that new theoretical constructs emerge in the process of solving important applied problems. Here are two examples.

In 1965, Tikhonov was approached by Prof. D.Ya. Martynov, director of the Shternberg State Astronomical Institute, who asked for help in the elaboration of stable numeric methods to interpret the data obtained in the observation of double eclipsing systems. This

work triggered further research. Jointly with his followers A.V. Goncharskii, V.V. Stepanov, A.G. Yagola, and A.M. Cherepashchuk, Tikhonov developed a system of interpretation of a priori information, such as monotony, convexity, and the number of maximums in the required solution. So, both mathematical physics and an important sphere of astronomy achieved new successes.

When he was director of the Institute of Applied Mathematics, Tikhonov for many years supervised the activities of a group of researchers: V.Ya. Arsenin, A.Kh. Pergament, N.A. Marchenko, and V.B. Mitrofanov, who studied inverse problems related to mathematical processing and interpretation of experimental data on plasma diagnostics. Computational tomography was another area of investigation. Tomography, that is, the restoration of a 3D object based on a set of its cross sections, is a major achievement in medical diagnoses in the 20th century. The Tikhonov theory of ill-conditioned problems constitutes the mathematical basis of this cutting-edge computational technology.

Note that the researchers A. Cormack (United States) and G. Hounsfield (Britain) won the 1979 Nobel Prize for the elaboration of computer tomography. Yet again, in 2003, another tomography-related work received the Nobel Prize in Physiology or Medicine, which went to P. Mansfield (Britain) and P. Lauterbur (United States) for research into magnetic-resonance tomography. We can only deplore the fact that the vast mathematical material worked out by the Tikhonov scientific school could not be incorporated in our national medical technology.

Scientific and organizational activities. Academician Tikhonov was not only a world-famous mathematician but also a brilliant organizer and leader of science. It became especially evident when he worked at the Institute of Applied Mathematics. Its predecessor, the Division of Applied Mathematics at the Institute of Mathematics of the USSR Academy of Sciences, had been formed on the basis of two teams of scientists; one team, which was directed by Keldysh and dealt with ballistics and celestial mechanics, was from the Institute of Mathematics, and the other, headed by Tikhonov and with substantial experience in gas dynamics, was made up of the researchers of Laboratory no. 8 of the Institute of Geophysics. Creative cooperation and fruitful joint work of these two outstanding mathematicians and organizers of science largely shaped applied mathematics and its applications in the Soviet Union.

Speaking about a multifaceted personality, one usually pays more attention to the qualities one has oneself and especially values in others. Here is an interesting account Tikhonov made of Keldysh:

One of the most remarkable qualities of M.V. Keldysh was his capacity to grasp the out-



M.V. Keldysh, A.N. Tikhonov, and A.A. Samarskii with the Institute of Applied Mathematics' veterans of World War II, 1975.

look of the problem under investigation and, at the same time, to immerse himself in its details. This was exactly what enabled Mstislav Vsevolodovich to initiate a number of major state scientific and technological programs; he could identify and present them so that they would appear clearly feasible.

At the same time, he could firmly decline a task with a vague outlook whose solution could not offer a good payoff. This latter feature is no less important for an executive and organizer than the readiness to do whatever is proposed [4, p. 202].

The newly organized Institute of Applied Mathematics had a number of divisions: gas dynamics (headed by K.A. Semendyaev), heat transfer (I.M. Gel'fand), mathematical physics (A.A. Samarskii), mechanics (D.E. Okhotsimskii), programming (A.A. Lyapunov), and neutron transfer (E.S. Kuznetsov). Looking through the reports on the activities carried out in the first years of the existence of the institute, one feels amazed at the scope of the problems tackled, the creative energy of the staff, and the lucidity of the leadership who could see the strategic outlook. And it all began in 1954, when the Strela computer, performing 1000 operations per second, was installed at the institute (now the staff have PCs many times more powerful and with enormous storage capacity.)

When Tikhonov was director of the institute (1978– 1989), it participated in many major scientific and technological projects, including the space shuttle Energiya-Buran system, one of the highest achievements of Soviet science and technology. In developing Buran, the designers used traditional techniques of airplane and rocket building. The main attention was given to the vehicle design, resistance, aerodynamics, engines, etc. However, Buran was a brainchild of a new technological era. Its maiden flight was to be (and in fact was) under automatic control. All its systems were to be computer operated. Computerized complexes were to be developed to be installed onboard the shuttle and on land to control all stages of the flight, from the launcher to deorbiting and landing at the prescribed point. These computerized complexes could only operate efficiently with appropriate software whose scope and complexity rivaled that of the hardware part of the project. It was for the first time that the researchers faced such an enormous challenge that the top project managers could not foresee. They had so many problems to address, such as the engines, that the software issue remained unattended. When its turn came at last, the situation already seemed catastrophic.

President of the USSR Academy of Sciences, Academician A.P. Aleksandrov thought that only the creation of a specialized institute with about 1200 programmers could resolve the problem successfully. Tikhonov got down to business, defined the problem,

organized, and did all the work in a short space of time with as few as a dozen highly skilled specialists.

Tikhonov was known for his tact, intellectuality, and careful consideration of decisions to be taken. But once a decision was made, he was prepared to show maximum persistence and exercise cast-iron will to implement it. This is why he succeeded in so many undertakings.

Our clients and colleagues often recall the meetings in which Tikhonov took part and where very important decisions were taken. Tikhonov was noted not only for his precise definitions and forethought. When the discussion ended with an agreement on the issue under examination, the next step was to define a course of actions to implement the decision. At this point, Tikhonov used to present the corresponding draft documentation already prepared. He always foresaw the trend of the debates and calculated many moves ahead.

He had the same serious and careful attitude to scientific seminars, conferences, and schools of young researchers. He attended all meetings and listened with much attention to what was said, he asked questions on the subject matter of each report even if the topic discussed was not in the sphere of his scientific interest. He even joked: if the listener has no questions to the presenter after the report, they both have wasted their time in vain.

Tikhonov had no special liking for references to high authorities and echelons of power made during discussions. He used to say: if you have substantial arguments, just come out to the blackboard with chalk in your hand and show them. His seriousness, good will, and, at the same time, stringent requirements on science and the work of his staff created a very special atmosphere at the institute and called for the highest standards of scientific activity.

MSU Faculty of Computational Mathematics and Cybernetics. One of the biggest projects of national importance implemented by Tikhonov was the creation of the Faculty of Computational Mathematics and Cybernetics (FCC) at MSU and the expansion of a network of such faculties to other universities over the country. This was a timely and far-seeing initiative that determined the level of national applied mathematics and computer technologies for many years to come.

In 1960, Tikhonov headed the Department of Computational Mathematics at the Faculty of Mechanics and Mathematics. In the late 1960s, this department became the biggest at the faculty, producing about 100 graduates every year. However, the upsurge of computers and introduction of computer-based technologies in science, industry, military science, and control systems made it necessary to step up these activities. Tikhonov was the first to understand and defend the necessity of training a great deal more new-generation mathematicians, says D.P. Kostomarov, a RAS cor-

responding member and Tikhonov's student and successor as the FCC dean.

Tikhonov proposed the creation of a new faculty from scratch. He was actively supported by outstanding mathematicians—Academicians A.N. Kolmogorov and Aleksandrov—who wrote a letter to the then university rector, Academician I.G. Petrovskii. The support given by Keldysh was also very important. Tikhonov recalled:

Mstislav Vsevolodovich took a great deal of interest in this idea; we discussed various aspects of this plan many times. It is significant that he prioritized the issue of the scientific team of the future faculty and discussed possible candidates. It was only after the personal selection of candidates had been agreed upon that Mstislav Vsevolodovich gave his full support to the initiative.

In fact, Tikhonov succeeded in involving many brilliant scientists in the work at the new faculty: A.A. Samarskii, Yu.V. Prokhorov, L.S. Pontryagin, S.V. Yablonskii, O.B. Lupanov, S.S. Lavrov, Yu.B. Germeier, V.V. Rusanov, M.R. Shura-Bura, L.N. Korolev. V.A. Il'in, M.M. Khapaev, Yu.L. Gaponenko and, some time later, D.P. Kostomarov came from the Department of Mathematics of the Physics Faculty. On September 1, 1970, classes began in all five years of the new faculty. Tikhonov was its dean from the moment of its establishment until 1990.

The large-scale and deep-laid strategy of the new faculty produces a great impression. Some of the teachers were invited only after the preparation and publication of their lecture courses. The printing of some of the courses reached 60 000 or more copies, since they were printed for the whole country and not just for one faculty. Some of the training courses elaborated in Tikhonov's time became classic; they were translated into many languages and are still used by the students of the faculty. However, as Tikhonov foresaw, computer science and applied mathematics advance impetuously. Once again it becomes necessary to update the content of the training courses for computer specialists and adopt new strategies. The experience of Tikhonov to form a new educational space in the forefront of computer sciences and technologies seems invaluable.

Note at what level the decision was made to create a new faculty. This decision was taken by the USSR Council of Ministers and the Defense Department of the Central Committee of the Soviet Party. Issues related to the training of computer specialists are dealt with at the highest level in many countries. The US Congress, for example, set up a special commission on teaching computer sciences that invites highly skilled experts to monitor education in this area and periodically review its content. This is not surprising: the entire innovational sphere of the economy and the present and future of the country depend on the situa-

tion in the information and communications system and on the qualification of its workers. The issues of education in this area should probably be examined on the same high level as it had been in Tikhonov's time.

Educational activities. Tikhonov was an outstanding educator. He taught several generations of specialists in applied mathematics, having defined the educational paradigm in this area for many decades to come.

His educational activities continued over half a century. Since 1927, in parallel to a postgraduate course at the MSU Research Institute of Mathematics and Mechanics, he worked as a math teacher in a secondary school in the Sokol'niki district of Moscow. In 1929, he began teaching at the university. Between 1933 and 1970, he taught at the Department of Mathematics at the MSU Faculty of Physics. From 1970 to 1981, Tikhonov headed the Department of Computational Mathematics and then, until 1990, the Department of Mathematical Physics at the FCC faculty. Those lucky enough to attend his lectures say that he lectured in a calm, low voice, keeping a low-key profile. The audience, however, were enchanted by the clarity, precision, and transparency of his exposition.

Tikhonov fully appreciated the potential of university science, which made it possible to secure an influx of young scientists, search for talented and enthusiastic students, and develop scientific schools. He always followed the Humboldt principle of continuance of science and education at the university.

New scientific disciplines grew from specific applied problems, which were generalized and deeply analyzed. This was how Tikhonov created several scientific schools that became known worldwide. One of them is the school of Academician Il'in, whose scientific activities began with the research into the convergence of bilinear series, a problem he had to tackle when he attempted to solve the thermal conductivity equation. This scientific school made outstanding discoveries in the theory of electromagnetic waves diffraction, spectral theory, and mathematical physics.

The theory (and school) of diffraction was formed by Prof. A.G. Sveshnikov, who for many years headed the Department of Mathematics at the MSU Physics Faculty, with the active participation of Tikhonov. This work began with the generalization of the results obtained by Tikhonov and Samarskii, who studied the excitation of oscillations in waveguides and their transition to irregular waveguides with an anisotropic filling and a complex-form surface. Tikhonov, Sveshnikov, V.I. Dmitriev, and A.S. Il'inskii were awarded the 1976 State Prize for outstanding results in computational electrodynamics and the theory of radiating-system synthesis.

The scientific school founded by the Profs. A.B. Vasil'eva and V.F. Butuzov (the current head of the Department of Mathematics at MSU Physics Faculty) gained a wide recognition in the world. This



Presidium of the ceremonial meeting dedicated to the 20th anniversary of the Institute of Applied Mathematics. In the foreground: A.N. Tikhonov and M.V. Keldysh, 1986.

school grew from a work by Tikhonov published in 1950 and dedicated to a system of equations with the initial conditions and the small parameter with the highest derivative:

$$\mu \frac{dz}{dt} = F(z, y, t), \quad \frac{dy}{dt} = f(z, y, t).$$

This type of equation emerged in the course of solving a specific applied problem of physical chemistry. However, the investigation of the so-called Tikhonov systems opened a new page in asymptotic analysis. It turned out that the area of application of singularly disturbed differential equations is very wide: from ecology and solid-state physics to mathematical biology and chemical kinetics.

Here is one more important feature of the Tikhonov's creative style. He always tried to depart from the essence of the problem and not from a method he knew or liked most. The vigorous progress of computer hardware has often caused (and still does) euphoria among the researchers, who think that they can calculate almost anything. Tikhonov, however, said many times that the important thing was not the volume of calculations but the correct way of stating a problem, the understanding that the researcher comes to in the process of solving this problem, and the exact answer to the question raised.

Tikhonov worked a lot on new textbooks, which was an important part of his educational activities. The year of 1951 saw the first print of the remarkable textbook *Equations in Mathematical Physics* by Tikhonov and Samarskii. It was a voluminous and clearly written book; on the one hand, it reflected the authors' understanding of modern mathematical physics, and, on the other, it became an ideal key that anyone could use to open the door of science.



Academicians P.S. Aleksandrov, I.G. Petrovskii, and A.N. Tikhonov in a lecture room of the MSU Faculty of Computational Mathematics and Cybernetics, 1973.

Tikhonov graduated from secondary school externally, without attending classes, and so, for the rest of his life, he paid much attention to book reading and self-instruction of students. He believed that a good textbook does not need additional explanation on the part of the teacher. All his books met this stringent requirement. The textbook of mathematical physics, which he wrote jointly with Samarskii, became an important event in the world of mathematics. It was republished several times and translated into many languages. The seventh edition was published in Russia in 2004 in the series "Classic University Manuals" with some additional material reflecting the results obtained in recent years by Tikhonov's students and the students that descended from them.

In the late 1950s, Tikhonov proposed the creation of a series of new textbooks on mathematics for the faculty of physics. This series "A Course of Higher Mathematics and Mathematical Physics" (edited by Tikhonov, Il'in, and Sveshnikov) became classic. For the new series, Tikhonov wrote the *Theory of Complex Variable Functions* together with Sveshnikov and *Differential Equations* with Vasil'eva and Sveshnikov. Tikhonov and Arsenin coauthored *Solution of Ill-Posed Problems*, which also won worldwide recognition.

In his scientific and organizational activities, Tikhonov strived to define the key priorities and concentrate the forces on their implementation. One such national-level priority since 1968 was teaching mathematics at secondary school. That year, Academician Kolmogorov and Academician of the USSR Academy of Pedagogical Sciences A.I. Markushevich, initiated the introduction of a new curriculum aimed at a fundamental revision of the ideology and content of the school course of mathematics. It was based on the transition to a multiple theoretical concept: they proposed

replacing the demonstrative approach—close to physics—with more abstract concepts.

Life has shown that this led to a sharp fall in the level of mathematics in school graduates. Tikhonov made enormous efforts to change the situation and elaborate new schoolbooks. On his initiative, a commission for a new reform was set up at the Branch of Mathematical Sciences of the USSR Academy of Sciences. Its members were Academicians Tikhonov, I.M. Vinogradov, A.V. Pogorelov, and Pontryagin. Jointly with representatives of the RSFSR Education Ministry, Tikhonov formed a team to write pilot textbooks. Algebra schoolbooks were to be written by Sh.A. Alimov (Moscow State University), Yu.M. Kolyagin (Research Institute of Schools), M.I. Shabunin, and Yu.V. Sidorov (Moscow Institute of Physics and Technology); and geometry schoolbooks, by L.S. Atanasyan (Lenin Pedagogical Institute), E.G. Poznyak, Butuzov, and S.B. Kadomtsev (Moscow State University).

Tikhonov, who was the ideologist of the new text-books and one of the initiators of revision in teaching mathematics, dedicated all his energy to the new task. His efforts bore fruit: at the national best textbook contest, the new textbooks prepared under his guidance took the lead. It should be noted that Tikhonov's attitude to the national experience of teaching was extremely considerate and careful: in 1980 he initiated the republishing of the evergreen geometry textbook by A.P. Kiselev, written at the beginning of the 20th century. The group of authors formed by Tikhonov is still working on new textbooks, in particular, for schools where mathematics is the main subject. Millions of students in Russia and abroad use these texts in their studies.

In the Soviet Union, school education was considered a matter of national importance. The Central Committee of the Communist Party set up a commission on the reform of secondary schooling, which included five members of the Politburo and in which Tikhonov represented the Branch of Mathematical Sciences of the USSR Academy of Sciences. His basic principle was to preserve what had already been achieved, and his favorite saying was "better always vanquishes good."

On one of his jubilees, Tikhonov was given a present symbolizing his scientific activities—a picture of a cube with the "Tikhonov brick" (one of the fundamental notions of general topology) on one face, the regularizing functional used in solving ill-conditioned problems on another, and the boundary function emerging in asymptotic decomposition in Tikhonov systems on the third. Almost everyone present was saying how many things the picture did not reflect because it did not show some of the cube faces. They all expressed their deep respect and love. Unfortunately we do not have the space to mention many of Tikhonov's fundamental results, his outstanding students, or write about the development of his scientific ideas in his final years.

Nevertheless, we hope that the next generation of researchers will remember this great mathematician and our fellow compatriot—Andrei Nikolaevich Tikhonov.

His long life was full of achievements and outstanding results; present and future generations can draw lessons from it. It is a vivid example of the role of a talented personality devoted to science in the development of research and his country.

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