MEMBERS OF SCIENTIFIC COMMUNITY

Nikolai Alekseevich Izobov

(A tribute in honor of his 60th birthday)



On January 23, 2000, Nikolai Alekseevich Izobov, Academician of the National Academy of Sciences of Belarus, Professor, Doctor of Sciences in physics and mathematics, celebrated his 60th birthday.

Nikolai Alekseevich Izobov was born in the village of Krasyni, Vitebsk region. In 1958 he left the Yanovichi secondary school as a gold medalist and entered the Department of Physics and Mathematics of the S. M. Kirov Vitebsk State Pedagogical Institute. Having completed his freshman and sophomore years and, after a one-year leave, the third year, in September, 1962 he was transferred to the third-year studies at the Department of Mathematics of the V. I. Lenin Belarus State University. Here, supervised by Yu. S. Bogdanov, Nikolai Alekseevich Izobov began his investigations in the asymptotic theory of ordinary differential equations, in particular, in the analysis of basic properties of lower Perron exponents of solutions to linear differential systems. At the All-Union competition of students' scientific papers, his results won him the medal "For the Best Student

Scientific Work" of the Ministry of Higher and Special Education of the USSR.

Izobov graduated from the university in December, 1965 and entered post-graduate studies in January, 1966. He wrote his Ph.D. thesis under Bogdanov's supervision and defended it in November, 1967. In January, 1969, Academician N. P. Erugin invited him to the position of Vice Editor-in-Chief of the All-Union journal "Differentsial'nye Uravneniya," where he worked for twelve years. Izobov defended the D.Sc. thesis "A Remark on the Theory of Lyapunov Characteristic Exponents of Linear and Quasilinear Differential Systems" (the summary was published in [33]) in Leningrad University in 1979 and was elected Associate Member of the Academy of Sciences of the BSSR in 1980. Since November, 1980 he has been working at the Institute for Mathematics as a senior researcher (1980–1986), the Head of the Laboratory of Stability Theory (1986–1993), and the Head of the Division of Differential Equations (1993–present). In 1994 he was elected Academician of the Academy of Sciences of Belarus.

Izobov's scientific research comprises five basic directions: the theory of Lyapunov characteristic exponents, the theory of stability with respect to the linear approximation, Coppel–Conti linear systems, Emden–Fowler equations, and Pfaff linear systems.

THE THEORY OF LYAPUNOV CHARACTERISTIC EXPONENTS

Already in the first papers [1, 2, 6, 7] (see also [61]) dealing with the investigation of basic properties of lower Perron exponents, Izobov, in particular, showed that the lower exponents of almost all solutions of a linear differential system with piecewise continuous bounded coefficients coincide with their maximum value and that there exist systems whose lower exponents form a set of positive Lebesgue measure.

In [18], Izobov proved that the main estimate of the Bol' freezing method, namely, the Alekseev-Vinograd estimate of the higher exponent of a linear nonstationary system, is sharp and obtained refinements and an integral version of this estimate for two-dimensional [16–19] and n-dimensional [22, 43] linear systems.

In the theory of transformations of linear systems, he proved general theorems [8, 20] on the reduction of a linear system by a generalized Lyapunov transformation to a system whose coefficients have a weak variation in the sense of Persidskii and on the reduction of a linear system with a small norm δ of the derivative of the coefficient matrix by a Lyapunov transformation to an almost diagonal system such that the norm of the off-diagonal entries of the coefficient matrix does not

exceed $\operatorname{const} \times \delta^{1/(1+n)}$ (the estimate is sharp). An interesting analog of the Euler integration method for stationary linear systems was constructed by Izobov [22] for nonstationary linear systems. In [60, 68], together with Makarov, he solved Bogdanov's problem on the existence of proper linear systems that become improper after the multiplication of the coefficient matrix by a positive scalar parameter. The papers [24, 25, 28, 29] deal with linear periodic systems.

In the framework of the problem, going back to Perron, on the stability of Lyapunov characteristic exponents of a linear system under small perturbations, Izobov, together with Bylov, obtained a criterion for the stability of characteristic exponents [11, 12]. (The sufficiency of this criterion was proved by B. F. Bylov, R. É. Vinograd, D. M. Grobman, and V. V. Nemytskii in the monograph *Theory of Lyapunov Exponents and Its Applications to Stability Problems* [in Russian], Moscow, 1966.) He also derived a coefficient criterion [21] for the stability of characteristic exponents of a two-dimensional linear nonstationary system in terms of eigenvalues and eigenvectors of the coefficient matrix.

In [26, 27, 30, 31, 35, 46], Izobov introduced the minimal exponent of a linear system, defined as the greatest lower bound of higher exponents of linear systems with small perturbations and suggested an algorithm for the computation of the minimal exponent in the two-dimensional case as well as a lower bound in the *n*-dimensional case.

For the higher and lower exponential exponents of a linear system, introduced in [37, 39, 40, 42], Izobov [37] constructed formulas based on the Cauchy matrix of the system and time geometric sequences. These exponents were used in [40, 42] for solving the special Lyapunov problem on the exponential stability with respect to the linear approximation. Izobov also introduced [10] (see also [57]) the higher sigma-exponent of a linear system and suggested [10] an algorithm for the computation of it via the Cauchy matrix. Later, together with Barabanov, he completely described the properties of this exponent [41]. The exponential exponents and sigma-exponents are now referred to as Izobov exponents.

Izobov [79, 80, 83, 90, 119] completely described the mutual arrangement of the following exponents on the set of linear differential systems: characteristic (Lyapunov) exponents, exponential (Izobov) exponents, central (Vinograd) exponents, and general (Bol') exponents.

Izobov put much effort into the investigation of linear systems with exponentially decaying perturbations. In particular, together with Stepanovich, he singled out the class of linear systems whose characteristic exponents coincide with those of linear systems with generalized Grobman perturbations and justified the existence of linear systems that do not belong to this class [69, 71, 73, 75]. Moreover, he proved [76–78, 81] the upper stability of the higher exponent and the lower stability of the lower exponent for linear systems under the same perturbations and established the existence of Grobman spectral sets of positive Lebesgue measure. In [81, 84, 85], Izobov obtained sharp estimates for the difference of the characteristic exponents of the same type of the original and perturbed systems with exponentially decaying perturbations. We also note his papers [5, 6], as well as the papers [92, 94, 97, 103, 110] written together with Filiptsov, concerning the invariance of lower Perron exponents under exponentially decaying perturbations.

Izobov's other results in this direction can be found in [3, 6, 57, 100, 104–106, 111, 115, 116, 126, 130].

The notions introduced by Izobov, the new problems posed by him, and the new methods he developed largely influenced the modern theory of linear systems of ordinary differential equations and related topics.

STABILITY WITH RESPECT TO THE LINEAR APPROXIMATION

To study the exponential stability of a differential system with *m*-perturbations with respect to the linear approximation, Izobov [9] (see also [15]) introduced the *a priori m*-exponent $\Omega_m(A)$ and the constructive *m*-exponent $\Omega_m''(A)$ of a linear system $\dot{x} = A(t)x$, $x \in \mathbb{R}^n$, $t \ge 0$, of order m > 1. The negative values of these exponents treated as functions of the parameter *m* were completely described in [49, 50, 53, 91] with the help of uniformly convergent series. We note Izobov's results [56, 59] on the existence of differential systems with higher-order infinitesimal perturbations and with sets of characteristic and lower exponents of positive Lebesgue measure. Together with Volkov [64, 65, 67], he established some other properties of these sets.

Lyapunov posed problems of exponential stability with respect to the linear approximation for differential systems with infinitesimal perturbations of arbitrary order (the special problem) and given order m > 1 (the general problem). The special problem was solved by Izobov in [40, 42, 138]:

the zero solution of a differential system is exponentially stable under an arbitrary infinitesimal perturbation of higher order if and only if the constructive exponent $\Omega''_m(A)$ of the linearized system is negative for all m > 1. In the case of the stronger exponential stability, this criterion requires [39, 40, 42, 138] the negativeness of the exponential exponent of the linearized system, which is the common sharp exponential asymptotics of the solutions.

In the solution of the general Lyapunov problem in the joint papers [13–15], it was shown that the Vinograd condition $\Omega'_m(A) < 0$ for the linearized system is sufficient and the Izobov condition $\Omega''_m(A) < 0$ is necessary (see also [9]) for the exponential stability of a differential system with the linear approximation $\dot{x} = A(t)x$, $x \in \mathbb{R}^n$, $t \ge 0$, and an arbitrary *m*-perturbation, m > 1. In the noncritical case, where these exponents coincide, this gives the solution of the general problem. But Izobov [52] (see also [93]) showed that these conditions are not necessary and sufficient, respectively, for the exponential stability in the critical case. However, he gave a complete solution of this problem with respect to the diagonal linear approximation [52].

COPPEL-CONTI LINEAR SYSTEMS

Izobov, together with Prokhorova, carried out quite a comprehensive study [55, 62, 63, 66, 72, 74, 82, 86, 95, 102, 107, 117, 118, 139] of the structure of the Coppel–Conti sets L^pS and M^pS , $p \geq 1$, of asymptotically stable and unstable linear systems with, in general, unbounded coefficients. These systems determine the boundedness of all solutions or some solution of nonhomogeneous linear systems with the corresponding nonhomogeneous terms.

In particular, they solved the first Conti problem, which asks whether the sets L^pS and M^pS shrink as p > 0 increases and proved that the dependence of these sets on the parameter p is not left continuous. Izobov proved [72, 74] criteria for the coincidence of the sets L^pS and M^pS with their interiors and solved the second Conti problem on the coincidence of the set $\lim_{p\to+\infty} L^pS$ with its interior as a special case of a more general problem. Izobov, together with Conti and Prokhorova [86, 87], thoroughly studied the problem as to whether the original and perturbed systems with perturbations integrable on $[0, +\infty]$ simultaneously belong to the sets L^pS and their left and right limit sets at the point p = q.

EMDEN-FOWLER EQUATIONS

Izobov's investigations of these classical equations, which have the form $u^{(n)} = p(t)|u|^{\lambda} \operatorname{sign} u$, $n \geq 2, \lambda > 0, t \geq 0$, where p(t) is a function of constant sign, are related to the solution [44, 45, 48] of two Kiguradze problems on the necessity of well-known criteria for the existence of unbounded proper and vanishing Kneser solutions. He obtained a general integral criterion [44, 45] for the absence of unbounded proper solutions of an Emden–Fowler equation with $\lambda > 1$ and showed that this criterion cannot be improved. Izobov, together with Rabtsevich, proved [54, 58] the sharpness of the Kiguradze–Kvinikadze criterion for the existence of proper solutions. In [48], Izobov constructed a parametric integral criterion for the absence of Kneser solutions of the Emden–Fowler equation with $\lambda \in (0, 1)$ and proved that this criterion is sharp for n = 2. He, together with Rabtsevich, refined this criterion in [70] for $n \geq 3$, and proved its sharpness.

LINEAR PFAFF SYSTEMS

In the recent papers [113, 114, 120, 122, 123, 125, 129, 131, 136], Izobov, in particular, proved the existence of linear completely integrable Pfaff systems with a countable set of pairwise distinct Gaishun–Grudo characteristic sets of solutions and with a lower characteristic set (introduced in [114]) of positive plane Lebesgue measure. Moreover, he, together with A. S. Platonov, obtained [121, 124, 128, 133, 135, 137] other properties of lower characteristic sets and proved that almost all solutions of an arbitrary linear Pfaff system have lower characteristic sets coinciding with the so-called maximal set.

In conclusion, note that many of Izobov's results were reported in the reviews [23, 33, 89, 117, 132, 138, 139].

In addition to intensive scientific work, for many years Nikolai Alekseevich Izobov has been teaching at the Belarus University, and in 1995–1999 he was in charge of the Chair of Higher Mathematics. Since 1994 he has been the Chairman of the Commission of Experts in Mathematics (Higher Certification Committee, Belarus) and a member of the Bureau of the Division of Physics, Mathematics, and Computer Science (National Academy of Sciences, Belarus). He has supervised fourteen philosophy doctors and one doctor of sciences. He was awarded the Honorary Diploma of the Council of Ministers of Belarus.

For more than 20 years, Nikolai Alekseevich Izobov was Vice Editor-in-Chief of the All-Union journal "Differentsial'nye Uravneniya"; now he is Vice Editor-in-Chief of the journal "Trudy Instituta Matematiki NAN Belarusi," a member of the editorial boards of the journals "Differentsial'nye Uravneniya," "Vestsi NAN Belarusi," and "Memoirs on Differential Equations and Mathematical Physics."

Izobov is very responsive and selfless. Everybody who gets in touch with him personally or by correspondence, feels his benevolent, constructive attention. We wish Izobov good health, further success in his scientific and managerial activity, and new gifted students.

V. N. Abrashin, I. V. Gaishun, V. A. Il'in, I. T. Kiguradze, S. K. Korovin, L. D. Kudryavtsev, A. B. Kurzhanskii, A. A. Martynyuk, V. M. Millionshchikov, Yu. A. Mitropol'skii, V. A. Pliss, N. Kh. Rozov, A. A. Samarskii, A. M. Samoilenko, T. K. Shemyakina

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ABRASHIN et al.

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DIFFERENTIAL EQUATIONS Vol. 36 No. 1 2000

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