U.S.S.R.,Comp.Maths.Math.Phys.Vol.23,No.2,pp.80-86,1983
Printed in Great Britain

0041-5553/83 $10.00+0.00

NON-STATIONARY DISSIPATIVE STRUCTURES* IN A NON-LINEAR HEAT-CONDUCTING MEDIUM

G.G. ELENIN, S.P. KURDYUMOV and A.A. SAMARSKII

The eigenfunctions of the non-linear boundary value problem, defining the non-stationary dissipative structures of the modes with peaking in an elementary one-component heat conducting medium, are studied. The stability of the dissipative structures is examined.

The emergence of structures is non-linear systems of different kinds plays an important role in modern science, as indicated by the rapidly growing number of monographs, surveys, and international symposia /1—8/. A wide range of publications (see surveys /5,7/) is devoted by the study of stationary dissipative structures and wave processes in systems of diffusion equations. For the relevant mathematical problems, global solutions are found, defined for all $t\geqslant 0$ (t is the time).

The present paper studies the non-stationary dissipative structures forming in a non-linear heat-conducting medium due to a mode with peaking. Modes with peaking (see definition 1 below) were previously studied e.g., in /9/, as examples of the non-existence of solutions in the large (a solution exists only in the bounded time interval $t\in[0, \tau)$, $\tau<+\infty$). In the present paper, and in others in which the present authors have been involved, modes with peaking are regarded as the intermediate asymptotic form /10/ of an evolutionary process.

The interest in modes with peaking is due to a variety of unusual properties. It is shown in /11—13/ that the heating of the medium caused by a non-linear heat source and occurring in a mode with peaking can be of various kinds.

In the S- and LS-modes with peaking, the heating is accompanied by localization of the heat (see definitions 2,3) on the fundamental length. With the HS-mode the heating of the medium is by a wave. In /14,15/ the spectrum of the dissipative structures of the LS-mode is found, and the stability of a simple structure is proved, while the averaging method is used to study features of the establishment of the structures and of the heat waves of the HS-mode.

The effect of heat localization in the S- and LS-modes with peaking makes it possible, not only to maintain, over a finite time interval, and in a bounded domain of space, an arbitrarily large value of the temperature and the quantity of heat, in spite of the presence of thermal conductivity, but also to arrive at the existence of non-stationary dissipative structures. Notice that a localization effect of a different physical kind occurs in a heat-conducting medium with absorption /16,17/. In this case, in connection with the action of the volumetric heat drain, the temperature and quantity of heat are bounded at any instant in a bounded domain. Among localization effects of another physical and mathematical kind may be mentioned the example of localized stationary dissipative structures studied in /3/.

The present paper contains an extension of the problem on dissipative structures produced by modes with peaking, and is closely linked with /14/. Our main attention is focussed on the Cauchy problem for the equation of heat conduction, in which the thermal conductivity and the heat source are power functions of the temperature. We analyze in detail the similarity solutions, which on the one hand have properties of interest to us, and on the other hand, admit of relatively simple methods of analysis. This approach — in the spirit of modern computing experiments, combining traditional methods of analysis with numerical methods /18/ — allows us to bring into relief the main features of the problem, to introduce some new concepts, and then to see a way of defining wide classes of coefficients of the equations, retaining the properties of the solutions which are of interest.

The similarity solutions are found by integration of the non-linear boundary value eigenvalue problem. Its eigenfunctions, corresponding to the positive eigenvalues, determine the concrete form of the different similarity modes with peaking. The eigenfunctions corresponding to the negative eigenvalues determine the solutions existing in the large (ordinary modes). A feature of the similarity solutions of the LS-mode is that the boundary value problem for any eigenvalue has a definite number of qualitatively different eigenfunctions. These functions define the simple and complex heat structures.

The similarity modes with peaking are unstable with respect to small disturbances of the initial data, though, as will be shown below, this does not mean that, given any initial data, the space structure of the solutions at the asymptotic stage differs from the space structure of the similarity solutions. By an analysis of the structural stability, using similarity processing (definitions 4,5), we isolate the class of similarity solutions, asymptotically stable in a special norm, matched with their space-time structure (see Sects.7,8).

1. The simplest mathematical model of a non-linear heat-conducting medium is the equation

*Zh.vȳchisl.Mat.mat.Fiz.,23,2,380-390,1983

$$T_t=(K(T)T_x)_x+Q(T), \quad (1)$$

where the thermal conductivity is a differentiable function and satisfies the conditions

$$K(T)>0, \quad T>0, \qquad K(0)=0, \qquad \int_0^1 K(x)x^{-1}dx<+\infty, \quad (2)$$

while the heat source is a convex non-negative function, vanishing at zero temperature and satisfying the condition

$$\int_\delta^\infty Q^{-1}(x)dx<+\infty, \qquad \delta>0. \quad (3)$$

For Eq.(1) we pose the problem

$$T(x,0)=T_0(x), \qquad 0\leq x<+\infty, \qquad 0\leq T_0(x)<+\infty,$$

$$\lim_{x\to+\infty} T_0(x)=0,$$

$$T_x(0,t)=0, \qquad \lim_{x\to+\infty} T(x,t)=0, \qquad \lim_{x\to+\infty}[K(T)T_x]=0,$$

whose solution, where $T(x, t)=0$, may not have the smoothness following from the equation (see /19/). In this connection we shall distinguish two cases.

Problem A. The initial distribution is finite:

$$T(x,0)=\begin{cases} T_0(x), & 0\leq x<a, \\ 0 & x\geq a. \end{cases}$$

Problem B. The initial distribution is non-zero throughout the space:

$$T(x, 0)=T_0(x)>0, \qquad 0\leq x<+\infty.$$

Problem A consists in finding the temperature $T(x, t)$ in a segment of variable length $l=x_\Phi(t)$ and the law of motion of its boundary $x=x_f(t)$, $x_f(0)=a$ when the temperature and heat flux is continuous:

$$T(x_f(t), t)=0, \qquad K[T(x_f(t), t)]\,T_x(x_f(t), t)=0.$$

The fact that a finite rate of motion of the heat wave front can exist is mentioned in /20/. In /17, 21, 22/ it is shown that satisfaction of conditions (2) ensures the existence of a finite velocity of the front. The fact that we distinguish problems A and B is not of essential importance, and merely facilitates the analysis.

Definition 1. The heating process occurs in the mode with peaking if $\tau>0$ and $x_0\geq 0$ exist such that

$$T(x,t)<+\infty \qquad \forall t\in[0,\tau) \qquad \text{and} \qquad \lim_{t\to\tau} T(x_0,t)=+\infty.$$

The quantity τ is the peaking time /11,12/, while (3) is the necessary condition for a mode with peaking to exist.

Definition 2. In problem A there is heat localization due to the mode with peaking if $x_0\geq a$ exists such that

$$\lim_{t\to\tau} x_f(t)<x_0 \qquad \text{and} \qquad T(x,t)=0 \qquad \forall x>x_0, \quad t\in[0,\tau].$$

The minimum value of x_0 is the depth of localization.

Obviously, localization is impossible when the velocity of propagation of the heat wave front is infinite, i.e., when condition (2) is infringed. The localization effect defined above does not occur in problem B, though effective localization may exist in this problem.

Definition 3. In problem B there is effective heat localization due to a mode with peaking, if $T_0>0$ and $x_0>0$ exist such that, given any $t\in[0,\tau]$ and $x>x_0$, we have $T(x, t)\leq T_0$. The minimum value of x_0 for the given T_0 is the depth of effective localization.

Let us demonstrate the features of modes with peaking in the class of power functions:

$$K(T)=T^\sigma, \qquad Q(T)=T^\beta, \qquad \sigma>0, \quad \beta>1. \quad (4)$$

Conditions (2) and (3) hold in this case. The power functions admit the existence of similarity solutions. Notice that in /23/ all the classes of functions $K(T), Q(T)$ were found, admitting the existence of invariant solutions /24/, of which the similarity solutions represent a particular case.

2. The expression $T(x, t)=g(t, \tau)\theta(\xi, \tau)$, $\xi=x\varphi^{-1}(t, \tau)$, is a particular solution of the initial problem under condition (4), provided that $g(t, \tau)=(1-t\tau^{-1})^{-\gamma}$, $\varphi(t, \tau)=(1-t\tau^{-1})^{\alpha}$, $\gamma=(\beta-1)^{-1}$, $\alpha=0.5\gamma(\beta-\sigma-1)$, and function $\theta(\xi, \tau)$ is the non-trivial solution of the boundary value problem for the equation

$$(\theta^\sigma\theta_\xi')_\xi'-\frac{\alpha}{\tau}\xi\theta_\xi'-\frac{\gamma}{\tau}\theta+\theta^\beta=0. \quad (5)$$

The boundary conditions in problem A have the form

$$\theta_\xi'(0)=0, \qquad \theta(a)=0, \qquad \theta^\sigma(a)\theta_\xi'(a)=0, \tag{6}$$

and in problem B, the form

$$\theta_\xi'(0)=0, \qquad \lim_{\xi\to+\infty}\theta(\xi)=0. \tag{7}$$

Problems (5), (6), and (5), (7) are problems on the eigenvalues τ and determine the eigenfunctions $\theta(\xi,\tau)$.

The positive eigenvalue and corresponding eigenfunction define the similarity solution of the mode with peaking. A negative value of τ along with the eigenfunction define the similarity solution existing in the large (for any $t>0$).

3. Problem (5), (7) has the solution $\theta(\xi,\tau)$ for any τ, if the solution $\theta(\xi,\tau_1)$ exists for some τ_1 such that $\tau\tau_1>0$. Here,

$$\theta(\xi,\tau)=(\tau\tau_1^{-1})^{-\gamma}\theta((\tau\tau_1^{-1})^{-\varkappa}\xi,\tau_1). \tag{8}$$

If problem (5), (6) has a solution in the segment $\xi\in[0,a_1], \beta\neq\sigma+1$, then a solution exists, given by (8), in any other segment $\xi\in[0,a]$ and

$$\tau=\tau_1(aa_1^{-1})^\omega, \qquad \omega=\alpha^{-1}. \tag{9}$$

These assertions are easily proved by a direct check.

4. For solutions of problem (5) and (6) to exist the conditions $1<\beta\leqslant\sigma+1, \tau>0$ and $\beta>\sigma+3, \tau<0$ must be satisfied.

For, on the one hand, the principal term of the asymptotic form ($\beta\neq\sigma+1$) as $\xi\to a: \theta(\xi)\sim[-\alpha\tau\sigma^{-1}(a-\xi)]^{1/\sigma}$ has a meaning under the condition $\tau(\beta-\sigma-1)<0$; and on the other hand, on integrating (5) in the light of (6), we obtain the inequality

$$-0.5(\beta-\sigma-3)(\beta-1)^{-1}\tau^{-1}=\int_0^a\theta^\beta(\xi,\tau)d\xi\left[\int_0^a\theta(\xi,\tau)d\xi\right]^{-1}>0.$$

With $\beta=\sigma+1$ the solution of the problem is found by direct integration of (5) (see /11/):

$$\theta(\xi,\tau)=\begin{cases}\theta_0\cos^{2/\sigma}\left(\dfrac{\pi}{L_s}\xi\right), & 0\leqslant\xi<0.5L_s,\\ 0, & \xi>0,5L_s,\end{cases} \tag{10}$$

where $L_s=2\pi\sigma^{-1}(\sigma+1)^{1/2}, \theta_0=[2(\sigma+1)\sigma^{-1}\tau^{-1}(\sigma+2)^{-1}]^{1/\sigma}$. The quantity L_s is called the fundamental length.

Problem (5), (7) has no solutions for $1<\beta\leqslant\sigma+1$. The necessary condition for solutions to exist such that

$$\int_0^\infty\theta(\xi,\tau)d\xi<+\infty,$$

is $\sigma+3>\beta>\sigma+1, \tau>0$. For, the asymptotic form of the solutions as $\xi\to+\infty$ is

$$\theta(\xi,\tau)=c\xi^{-2(\beta-\sigma-1)^{-1}}, \qquad c=\text{const}>0, \tag{11}$$

and

$$\int_0^\infty\theta^\beta(\xi,\tau)d\xi\left[\int_0^\infty\theta(\xi,\tau)d\xi\right]^{-1}=-(\beta-\sigma-3)(\beta-1)^{-1}\tau^{-1}>0.$$

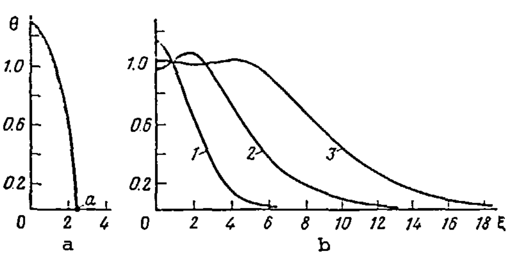

Fig.1

5. Qualitative and numerical analysis* shows that, with $1<\beta<\sigma+1$ and arbitrary $a>0$, problem (5), (6) has a unique solution $\theta(\xi,\tau)$ corresponding to the unique eigenvalue $\tau=\tau(a)$. The time τ decreases as a increases in accordance with (9). The eigenfunction $\theta(\xi,\tau)$ is monotonic for any fixed $\tau>0$ (see Fig.1,a). With $\beta=\sigma+1$, problem (5), (6) has a one-parameter family of non-trivial solutions, existing in the unique interval $a=0.5L_s$ (see (10)). The free parameter of the family is the peaking time $\tau>0$. With $\sigma+1>\beta>\sigma+3$ problem (5), (6) has no non-trivial solutions. In the same band, problem (5), (7) has for every positive τ a definite number N of eigenfunctions $\theta_i(\xi,\tau)$, $i=1,2,\ldots,N$. Their number is equal to the number of zeros in the solution $y(x)$ of the following linear problem (it is established by means of a numerical experiment):

$$y''-0.5(\beta-\sigma-1)xy'+(\beta-1)y=0, \qquad y'(0)=0. \tag{12}$$

The solution of this problem is

$$y(x)=c_0\Phi(-(\beta-\sigma)(\beta-\sigma-1)^{-1}, 0.5, 0.25(\beta-\sigma-1)x^2), \qquad c_0\neq0, \tag{13}$$

where $\Phi(a,b,\xi)$ is the degenerate hypergeometric function /25/, and c_0 is an arbitrary constant. The number of zeros of solution (13) is found with the aid of the function $[z]$ (the integer part of the number z) from the expression

$$N=[z-[[z]z^{-1}]]+1, \qquad z=(\beta-1)(\beta-\sigma-1)^{-1}.$$

The eigenfunctions are such that

$$\int_0^\infty \theta_i(\xi,\tau)\,d\xi<+\infty.$$

In Fig.1,b we give the three eigenfunctions for the case $\beta=3.67$, $\sigma=2.0$ and $\tau=(2.67)^{-1}$ ($\theta_1(\xi,\tau)$ is curve *1*, $\theta_2(\xi,\tau)$ is curve *2*, $\theta_3(\xi,\tau)$ is curve 3). With $\beta>\sigma+3$ solutions of problem (5), (7) exist for any $\tau>0$: $\theta_i(\xi,\tau)$, $i=1,2,\dots,N$. In this case

$$\lim_{\xi\to+\infty}\int_0^\xi \theta(\eta,\tau)\,d\eta=+\infty$$

(see (11)). Moreover, in the same range of parameters β,σ, problem (5), (6) has a unique non-trivial solution $\theta(\xi,\tau)$, corresponding to the unique number $\tau=\tau(a)$. As a increases, $|\tau|$ increases in accordance with (9). The eigenfunction is monotonic.

6. The expression $T(x,t)=g(t,\tau)\theta(\xi,\tau)$ is the exact solution of our problems, if the initial data are similarity data: $T(x,0)=\theta(x,\tau)$. With $1<\beta<\sigma+1$ and any $a>0$, there is a unique initial temperature distribution $\theta(x,\tau(a))$, determining a unique similarity heat wave of the so-called *HS*-mode with peaking. The peaking time is given by the length of the segment a. With $\beta=\sigma+1$, in the unique interval $a=0.5L_t$ there is a one-parameter family of initial data $\theta(x,\tau)$, defining non-stationary dissipative structures of the S-mode with peaking, localized in the fundamental length. The peaking time of the structure is determined by the amplitude of the initial distribution θ_0 (see (10)).

The similarity initial data $T(x,0)=\theta_i(x,\tau)$, $\tau>0$, $\beta>\sigma+1$, define N similarity solutions of the mode with peaking, having a qualitatively different space structure and existing during the same time interval $\tau>0$. The similarity solution, corresponding to $\theta_1(\xi,\tau)$ has a maximum at $\xi=0$. The half-width of this solution $x_h(t)$: $T(0,t)=2T(x_h(t),t)$ tends monotonically to zero as $t\to\tau$. The similarity solutions corresponding to $\theta_i(\xi,\tau)$, $i\geq2$, have i local extrema. In these solutions, all the local extrema move towards the centre of symmetry $x=0$ and at the instant $t=\tau$ focus at it, while leaving in space the trace (the limiting temperature distribution)

$$T_i(x)=\lim_{t\to\tau}T(x,t)=c_i x^{-2(\beta-\sigma-1)^{-1}}, \tag{14}$$

where $c_i>0$ is a constant, dependent on $\tau>0$ and on the number i of the eigenfunction. The existence of limiting distribution (14) reveals the effective localization in the similarity solutions of the so-called *LS*-mode with peaking /14/. The similarity solutions corresponding to the first eigenfunction determine the simple dissipative structure, while the higher eigenfunctions determine the complex dissipative structures.

With $\beta>\sigma+3$, along with simple and complex heat structures of the mode with peaking, there is a similarity heat wave existing for any $t>0$. The heat wave amplitude decreases with time. For any $a>0$, there is a unique initial distribution $\theta(x,\tau(a))$ and a unique value $\tau(a)<0$, defining this wave.

Table 1

Problem	$1<\beta<\sigma+1$	$\beta=\sigma+1$	$\sigma+1<\beta<\sigma+3$	$\beta>\sigma+3$
A B	*HS*-mode with peaking	*S*-mode with peaking	*LS*-mode with peaking	Ordinary *HS*-mode *LS*-mode with peaking

In Table 1 we classify the solutions of problems A and B both according to the time-variation of the temperature (ordinary modes, existing in the large, and modes with peaking), and according to the time-variation of the half-width. Following /11, 12/, we call the similarity solutions with monotonically decreasing, fixed, and monotonically increasing, half-width, respectively the *LS*-, *S*-, and *HS*-modes.

7. The similarity solutions of modes with peaking are unstable with respect to the initial data. A small disturbance of the initial distribution leads to a small variation of the peaking time. This in turn leads to an arbitrarily large difference between the solutions, starting from a certain instant close to the instant of peaking. This type of instability does not mean that always, in the case of non-similarity initial data, the space structure of the solutions at the asymptotic stage differs from the space structure of the similarity solution.

Definition 4. Let $T(x,t)$ be the solution of the original problem. The transformation of the solution in accordance with the expression

$$\bar\theta(\xi,t)=T_m^{-1}(t)\,T(\xi f(T_m(t)),t), \tag{15}$$

where $f(z)=z^{-0.5(\beta-\sigma-1)}$,

$$T_m(t)=\begin{cases} \text{either} & T(0,t), \\ \text{or} & \max\limits_{0<x<+\infty} T(x,t), \end{cases}$$

is called similarity processing, and $\theta(\xi, t)$ is the similarity form of the solution /14/.

Applying transformation (15) to the similarity solution, we obtain the normalized eigenfunctions

$$\bar{\theta}(\xi)=\begin{cases} \text{either} & \theta^{-1}(0)\theta(\xi,\tau), \\ \text{or} & [\max\limits_{0<\xi<+\infty} \theta(\xi,\tau)]^{-1}\theta(\xi,\tau). \end{cases}$$

A normalized eigenfunction is a stationary similarity processing (15).

Definition 5. We shall say that the similarity solution of a mode with peaking has a stable space structure if the normalized eigenfunction defining this solution is stable and stationary among the similarity representations of the solution $\theta(\xi, t)$.

8. Analytical methods and a numerical experiment /14, 26/ show that, in the range $1<\beta<\sigma+1$ the similarity solutions are stable in the sense of definition 5. In Fig.2 ($0<t_1<t_2<t_3<\tau$) we give the results of a numerical computation with similarity processing. The heating is realized by the similarity heat wave of the HS-mode with peaking. With $\beta=\sigma+1$ the similarity solution of the S-mode is stable. In the fundamental length a symmetric stable non-stationary dissipative structure of the mode with peaking is formed and an effect of heat localization occurs. In the parameter range $\sigma+3>\beta>\sigma+1$ the simple heat structure is stable. The complex structures are preserved during almost the entire peaking time. During a fairly short time directly before the instant of peaking, degeneration of the complex structure to a simple structure occurs (see Fig.3). With $\beta>\sigma+3$, there are two types of similarity solutions: the heat wave of the ordinary HS-mode and the heat structures of the LS-mode with peaking (see Table 1). The similarity solution of the ordinary HS-mode is unstable. The numerical experiment showed that, if the initial distribution is minorized by the temperature distribution of the similarity solution of the ordinary HS-mode, then the LS-mode with peaking occurs, or if it is majorized, then over a fairly long term a damped wave exists. In both cases the maximum temperatures in the initial and the similar distributions are the same (see Fig.4).

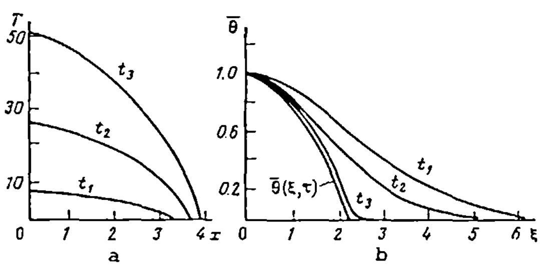

Fig.2

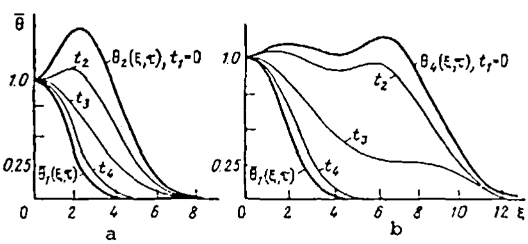

Fig.3

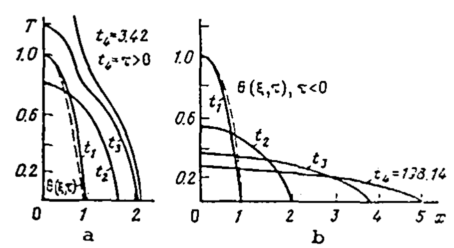

Fig.4

9. It has been shown above that problem A has no similarity solutions of modes with peaking in the range $\beta>\sigma+1$. However, by using the averaging method /14/, /26/, the comparison theorems /27 — 30/, and a computing experiment /11/, it can be shown that a heat localization effect in the sense of definition 2 occurs in the solutions of problem A, in spite of the absence of similarity solutions. Non-stationary and dissipative structures exist, localized in the fundamental length of the LS-mode. Many numerical experiments /14/ show that the temperature distribution in the structures of problem A, almost everywhere inside the localization domain, with the exception of the neighborhood of the front $x=x_\Phi(t)$: $T(x_\Phi(t), t)=0$ ($x_\Phi(\tau)$ is the fundamental length of the LS-mode), coincides with the temperature distribution in the similarity mode of problem B. An estimate of the fundamental length of the LS-mode with peaking is given in /13/.

10. Further studies have shown that modes with peaking and heat localization are typical for wide classes of thermal conductivity and a spatial heat source. An important role in determining these classes is played by the comparison theorems for parabolic (including degenerate) equations /27—36/, and by the construction of approximate similarity modes /37/. Important results of such work include a method for constructing, given $Q(T)$, the coefficient

$K(T)$ leading to heat localization in a mode with peaking. Applications of the localization effect are discussed in /38—44/. Interaction of heat structures in the multi-dimensional case has been studied numerically in /45,46/. The averaging method, proposed in /14, 26/, has proved useful when studying the transition into the similarity mode. Dissipative structures of modes with peaking in media with distributed parameters are considered in /47, 48/. The series of papers /49—54/ is devoted to the study of non-stationary structures in different mathematical models of non-linear media, including multi-component media. Effects of heat localization in problems with limiting modes are considered in /55/, where very complete references will be found.

REFERENCES

1. HAKEN H., Synergetics, Springer, Berlin, 1980.
2. GLENSDORFF P., and PRIGOZHIN I., Thermodynamic theory of structure, stability and fluctuations /Russian translation/, Mir, Moscow, 1973.
3. NIKOLIS G. and PRIGOZHIN I., Self-organization in non-equilibrium systems /Russian Translation/, Mir, Moscow, 1979.
4. ROMANOVSKII YU.M., STEPANOVA N.V. and CHERNAVSKII D.S., Mathematical models in biology (Matematicheskie modeli v biologii), Nauka, Moscow, 1975.
5. VASIL'EV V.A., ROMANOVSKII YU.M., and YAKHNO V.G., Similarity processes in distributed kinetic systems, Usp. fiz. Nauk, 128, No.4, 625-666, 1979.
6. KURDYUMOV S.P., Eigenfunctions of the heating of a non-linear medium and constructive laws for constructing its organization, Preprint IPMatem. AN SSSR, No.29, Moscow, 1979.
7. DIKANSKII A.S., Equations of diffusion with non-linear kinetics, Dep. at VINITI, No.1405-80 DEP, 1980.
8. HAKEN H., Synergetics, Springer, Berlin, 1980.
9. LEVINE H.A., Some nonexistence and instability theorems for solutions of formally parabolic equations of the form $Pu_t=-Au+F(u)$ Arch. Ration. Mech. and Analysis, 51, 371-386, 1973.
10. BARENBLATT G.I. and ZEL'DOVICH YA.B., Intermediate asymptotic behaviour in mathematical physics, Usp. mat. Nauk, 26, No.2, (158), 115-129, 1971.
11. SARMARSKII A.A., et al., Thermal structures and fundamental length in a medium with non-linear thermal conduction and a spatial heat source, Dokl. Akad. Nauk SSSR, 227, No.2, 321-324, 1976.
12. ZMITRENKO N.V., et al., Appearance of structures in non-linear media and non stationary thermodynamics of modes with peaking, Preprint IPMatem., Akad. Nauk SSSR, No.74, Moscow, 1976.
13. ZMITRENKO N.V., et al., Non-linear processes in dense plasma and features of thermodynamics of modes with peaking, Preprint IPMatem. Akad. Nauk SSSR, No.109, Moscow, 1976.
14. ELENIN G.G., and KURDYUMOV S.P., Conditions for complexity of organization of non-linear dissippative medium, Preprint IPMatem. Akad. Nauk SSSR, No.106, Moscow, 1977.
15. SAMARSKII A.A., et al., Heating of a non-linear medium in the form of complex structures, Dokl. Akad. Nauk SSSR, 237, No.6, 1330-1333, 1977.
16. MARTINSON A.K. and PAVLOV K.B., On the space localization of heat disturbances in the theory of non-linear conduction, Zh. vych. Mat. i mat. Fiz., 12, No.4, 1048-1053, 1972.
17. KALASHNIKOV A.S., On the influence of absorption on heat propagation in a medium with temperature-dependent heat conduction, Zh. vych. Mat. i mat.Fiz., 16, No.3, 689-696, 1976.
18. SAMARSKII A.A., Mathematical modelling and computing experiments, Vestn. Akad. Nauk SSSR, No.5, 38-49, 1979.
19. OLEINIK O.A., KALASHNIKOV A.S., and CHOU YU LIN, The Cauchy problem and boundary value problems for equations of the non-stationary porous flow type, Izv. Akad. Nauk SSSR, Ser, matem., 22, No.5, 667-704, 1958.
20. ZEL'DOVICH YA.B., and KOMPANEETS A.S., Theory of heat propagation with temperature-dependent heat conduction, in: Collection celebrating Acad. A.F., Ioffe's seventieth birthday (Sb. posvyashchennyi 70-letiyu akad. A.F. Ioffe, Izd-vo AN SSSR, Moscow, 1951.
21. KALASHNIKOV A.S., On quasi-linear degenerate parabolic equations with finite propagation rate of disturbances, in: Partial differential equations (Differents. urniya s chastnymi proizvodnymi), Nauka, Novosibirsk, 1980.
22. GALAKTIONOV V.A., On some properties of travelling waves in a medium with non-linear heat conduction and heat source, Zh. vych. mat. i mat. fiz., 21, No.4, 980-989, 1981.
23. DORODNITSYN V.A., Group properties and invariant solutions of the equation of heat conduction with source or sink, Preprint IPMatem. Akad. Nauk SSSR, No.57, Moscow, 1979.
24. OVSYUNNIKOV L.V., Group properties and invariant solutions of the equations of non-linear heat conduction, Dokl. Akad. Nauk SSSR, 125, No.3, 492-495, 1959.
25. BATEMAN H. and ERDELI A., Higher transcendental functions, 1960.
26. ELENIN G.G. and PLOKHOTNIKOV K.E., On a method for the qualitative study of non-dimensional quasi-linear equation of heat conduction with a non-linear heat source, Preprint IPMatem. Akad. Nauk SSSR, No.91, Moscow, 1977.
27. GALAKTIONOV V.A., et al., On comparison of the solutions of parabolic equations, Dokl. Akad. Nauk SSSR, 248, No.3, 586-589, 1979.

28. GALAKTIONOV V.A., et al., On an approach to the comparison of the solutions of parabolic equations, Zh. vych. Mat. i mat. Fiz., 19, No.6, 1451-1461, 1979.
29. GALAKTIONOV V.A., Conditions for ψ-criticality and methods of comparing the solutions of parabolic equations, Preprint IPMatem. Akad. Nauk SSSR, No.151, Moscow, 1979.
30. GALAKTIONOV V.A., Two methods of comparing the solutions of parabolic equations, Dokl. Akad. Nauk SSSR, 251, No.4, 832-835, 1980.
31. GALAKTIONOV V.A., et al., On unbounded solutions of semi-linear parabolic equations, Preprint IPMatem. Akad. Nauk SSSR, No.161, Moscow, 1979.
32. GALAKTIONOV V.A., et al., Asymptotic stage of modes with peaking and effective heat localization in problems of non-linear heat conduction, Differents. ur-niya, 16, No.7, 1196-1204, 1980.
33. GALAKTIONOV V.A., Some properties of the solutions of quasi-linear parabolic equations, Preprint IPMatem. Akad. Nauk SSSR, No.16, Moscow, 1981.
34. GALAKTIONOV V.A., et al., On unbounded solutions of the Cauchy problem for the parabolic equation $u_t=\nabla(u^\sigma\nabla u)+u^\beta$, Dokl. Akad. Nauk SSSR, 252, No.6, 1362-1364, 1980.
35. GALAKTIONOV V.A., On a boundary value problem for the non-linear parabolic equations $u_t=\Delta u^{\sigma+1}+u^\beta$, Differents. ur-niya, 17, No.5, 836-842, 1981.
36. SAMARSKII A.A., On some problems of the theory of differential equations, Differents. ur-niya, 26, No.11, 1925-1935, 1980.
37. SAMARSKII A.A., On new methods of studying the asymptotic properties of parabolic equations, Tr. Matem. in-ta Akad. Nauk SSSR, 158, 153-162, Moscow, 1981.
38. ZMITRENKO N.V., KURDYUMOV S.P., and SAMARSKII A.A., The possibility of using heat localization in modes of θ-pinch compression with peaking, Preprint IPMatem. Akad. Nauk SSSR, No.153, Moscow, 1980.
39. ELENIN G.G., et al., Heat inertia in dissipative structures, in: Study of hydrodynamic instability by numerical methods, IPM Akad. Nauk SSSR, Moscow, 1980.
40. ZMITRENKO N.V., and KURDYUMOV S.P., N and S-modes of compression of finite mass of a plasma and features of modes with peaking, Prikl. mekhn. i tekh. fiz., 1, 3-18, 1977.
41. ZMITRENKO N.V., et al., Localization of thermonuclear heat in a plasma with electronic heat conduction, Letter to ZhEFR, 26, No.9, 620-623, 1977.
42. SAMARSKII A.A., and KURDYUMOV S.P., Nonlinear processes in a dense plasma and their role in the problem of laser controlled thermonuclear fusion, in: Wave and gas dynamics (Volnovaya i gazovaya dinamika), No.3, Izd-vo MGO, Moscow, 1979.
43. ZMITRENKO N.V. and KURDYUMOV S.P., Compression and rarefaction modes of finite mass, admitting time conversion in a dissipative medium, Preprint IPMatem. Akad. Nauk SSSR, No. 39, Moscow, 1981.
44. SAMARSKII A.A., Numerical simulation in plasma physics, in: Comput. Methods Appl. Sci. and Engng., INRIA, North Holland Publ. Co., 1980.
45. KUDYUMOV S.P., et al., Interaction of heat structures, Preprint IPMatem. Akad. Nauk SSSR, No.77, 1978, Moscow.
46. KURDYUMOV S.P., et al., Interaction of dissipative heat structures in media with distributed parameters, Dokl. Akad. Nauk SSSR, 251, No.4, 836-839, 1980.
47. KURDYUMOV S.P., KURKINA E.S. and MALINETSKII G.G., Dissipative structures in non-linear media, Preprint IPMatem. Akad. Nauk SSSR, No.16, Moscow, 1979.
48. KURDYUMOV S.P., KURKINA E.S. and MALINETSKII G.G., Dissipative structures in an inhomogeneous non-linear hot medium, Dokl. Akad. Nauk SSSR, 251, No.3, 587-591, 1980.
49. GALAKTIONOV V.A., et al., Influence of burnup on heating localization and the formation of structures in a non-linear medium, Preprint IPMatem. Akad. Nauk SSSR, No.27, Moscow, 1979.
50. KURDYUMOV S.P., et al., Non-stationary dissipative structures in non-linear two-component media with spatial sources, Dokl. Akad. Nauk SSSR, 258, No.5, 1084-1088, 1981.
51. MALINETSKII G.G., On a class of mathematical models connected with self-organization, Preprint IPMatem. Akad. Nauk SSSR, No.192, Moscow, 1980.
52. KURKINA E.S., and MALINETSKII G.G., Some effects of self-organization in plasma physics, Preprint IPMatem. Akad. Nauk SSSR, No.122, Moscow, 1980.
53. KURDYUMOV S.P., KURKINA E.S. and MALINETSKII G.G., Matched heating modes in a dissipative medium with transport, Preprint IPMatem. Akad. Nauk SSSR, No.125, Moscow, 1980.
54. ZMITRENKO N.V. and MIKHAILOV A.P., Analytic methods of studying some non-linear problems of plasma physics. Method text-book (Analiticheskie metody issledovaniya nekotorykh nelineinykh zadach fiziki plazmy), TsNIII atominform., Moscow, 1981.
55. GALAKTIONOV V.A., et al., Heat localization in non-linear media, Differents. ur-niya, 17, No.10, 1826-1841, 1981.

Translated by D.E.B