THE SELF-MODELLING PROBLEM OF A HEAVY CURRENT DISCHARGE IN A PLASMA*

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1. A STUDY of the processes that occur when a heavy-current radiating discharge takes place in a plasma involves the solution of the set of equations of magnetic radiation hydrodynamics (MRHD). In general, a solution can only be obtained by using numerical methods. Examples of such solutions may be found e.g., in [1–3].

While the use of self-modelling solutions in this problem involves serious restrictions imposed by the self-modelling conditions, it nevertheless enables individual qualitative aspects of the process to be investigated, and the type of dependence of the process on problem parameters such as the electrical and thermal conductivity, discharge current etc. to be revealed.

In the present paper we examine self-modelling solutions in which the mass of plasma in the discharge is time-independent. It is shown that self-modelling solutions of this type only exist when the thermal conductivity is fairly high. The lower limit of the range of thermal conductivity variation within which the self-modelling solution exists is determined in some particular cases.

A T-layer [4] is shown to exist under certain conditions in the self-modelling solutions. The influence on its structure of the heat conduction process is examined.

Analysis of the self-modelling solutions is supplemented by computer evaluations for the complete set of equations of MRHD, in both the self-modelling and 'almost self-modelling' ranges of parameter variation.

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The most detailed attention is paid to self-modelling modes in which the total energy as well as the mass of the plasma is time-independent. The fact that the energy is fixed derives in the present problem, not from the fact that the system is conservative, but from the equality of the energy fluxes entering and leaving the system.

It may be noted that the self-modelling solutions obtained in this paper provide a good test for checking the accuracy of numerical methods for solving the system of equations of MRHD. In particular, they were used in [3] when developing and refining numerical methods.

2. We shall examine the separation in vacuo of the plasma formed by electrical fusion of a wire, and its interaction with the magnetic field of the natural currents (see Fig. 1). The approximation of non-linear heat conduction is used for the heat transfer processes.

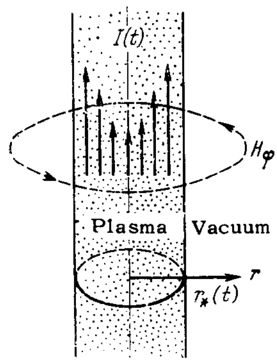

FIG. 1.

We assume that the length of the plasma filament is much greater than its diameter, and that there is axial symmetry; the problem is considered in the one-dimensional non-stationary approximation for an infinite cylinder.

Using Lagrange mass coordinates and the absolute Gaussian system of units, the relevant set of equations of magnetohydrodynamics is [5]

$$\frac{\partial v}{\partial t} = -r\frac{\partial p}{\partial x} - \frac{1}{c}\frac{j_z H_\varphi}{\rho}, \qquad \frac{\partial r}{\partial t} = v, \qquad \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \frac{\partial (rv)}{\partial x},$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{H_\varphi}{\rho r}\right) = \frac{\partial E_z}{\partial x}, \qquad E_z = \frac{c\rho}{4\pi\sigma}\frac{\partial (rH_\varphi)}{\partial x}, \qquad j_z = \sigma E_z,$$

$$\frac{\partial \varepsilon}{\partial t} = -p\frac{\partial (rv)}{\partial x} - \frac{\partial (rW)}{\partial x} + Q,$$

$$W = -\varkappa\rho r\frac{\partial T}{\partial x}, \qquad Q = \frac{j_z E_z}{\rho}, \tag{2.1}$$

$$p = \rho RT, \qquad \varepsilon = \frac{R}{\gamma - 1}T.$$

Here, t is the time, r is Euler's variable, ρ is the density of the medium, $x\,(dx = \rho r dr)$ is the Lagrange mass variable, v is the longitudinal component of velocity, p is the pressure, ϵ is the internal energy, T is the plasma temperature, H_ϕ is the azimuthal component of the magnetic field strength, E_z is the axial component of the electric field strength, j_z is the electric current density, σ and κ are the electrical and thermal conductivities respectively, Q is the Joule heat evolved per unit mass, W is the heat flux through one radian of azimuth, R is the gas constant, γ is the adiabatic exponent, and c is the velocity of light in vacuo; the time derivative is Lagrangian.

The simplest form of the equation of state is used.

The solution of the problem will be sought for a cylinder of unit height in the region $t \geqslant 0$, $0 \leqslant x \leqslant M$, where $M = \int_0^{r_*} \rho r\, dr = \text{const}$ is the mass of plasma in the discharge per unit height of plasma filament and per radian of azimuth, and $r_*(t)$ is the radius of the plasma-vacuum boundary. The boundary conditions for (2.1) are as follows: at the centre, with $x = 0$, the symmetry conditions are

$$v(0, t) = 0, \qquad H_\varphi(0, t) = 0, \qquad W(0, t) = 0, \tag{2.2}$$

while to the right of the plasma-vacuum boundary, with $x = M$ $(r = r_*(t))$

$$p(M, t) = 0, \qquad H_\varphi(M, t) = 2I(t)\,/\,cr_*(t), \qquad T(M, t) = 0; \tag{2.3}$$

$I(t)$ gives the variation with time of the total current in the discharge.

In the general case of MRHD, when the medium has non-linear heat conduction properties, the temperature on the material-vacuum boundary is non-zero. The condition $T(M, t) = 0$ is the limiting case, ensuring the absence of heat flow from the vacuum into the system.

Other types of right-hand boundary condition are possible for the heat functions, e.g., $W(M, t) = 0$, corresponding to the case of electronic heat conduction, or $W(M, t) = \sigma_c T^4$: when the plasma filament radiates like a black body

(σ_c is the Stefan–Boltzmann constant). The latter condition leads to additional restrictions in the self-modelling conditions obtained below.

To obtain the self-modelling solution, we consider the asymptotic stage in the plasma separation, when the influence of the initial data is no longer felt. Here, the initial diameter of the plasma filament can be neglected as compared with its dimensions at the asymptotic stage, and accordingly, the initial plasma density may be assumed infinite. This enables the number of definitive parameters in the problem to be reduced.

The electrical and thermal conductivities are assumed to be power functions of the temperature and density; to achieve greater generality when deriving the self-modelling conditions, explicit dependences of the conductivities on time are also introduced:

$$\sigma = \sigma_0 T^{k_0} \rho^{-q_0} t^{n_0}, \qquad \varkappa = \varkappa_0 T^{k_1} \rho^{-q_1} t^{n_1}. \tag{2.4}$$

It will also be assumed that $I(t)$ is a power function:

$$I(t) = I_0 t^m. \tag{2.5}$$

The case of fixed current, $m = 0$, is treated in detail below.

We shall seek the self-modelling solution of the set of equations (2.1), in which all the functions can be written in the form $F(x, t) = F_0 f(s) t^{n_f}$, where F_0 is a dimensional constant, $s = x/M$ is the self-modelling variable, proportional to the mass variable, and $f(s)$ is a dimensionless function of the self-modelling variable. Self-modelling solutions of this type were investigated in [6, 7].

Analysis shows that the self-modelling conditions reduce in this case to satisfying certain relationships between the problem parameters, i.e., the powers in (2.4) and (2.5):

$$m + 1 = \frac{2k_0 + 1 - n_0}{2(k_0 + 1) + 2q_0} = \frac{2k_1 - 1 - n_1}{2(k_1 + q_1)}. \tag{2.6}$$

It also follows from (2.6) that, if we assume fixed relationships (2.4) satisfying the second of equations (2.6), a solution with self-modelling properties can be guaranteed by a suitable choice of the current law (the quantity m in (2.5)).

For instance, if the current is increasing, $m > 0$, and there is no time dependence in (2.4) and (2.5) ($n_0 = n_1 = 0$), the self-modelling conditions (2.6) lead to the inequalities

$$q_0 \leqslant -0.5, \quad q_1 \leqslant -0.5.$$

This means that the electrical and thermal conductivities should increase with the density, whereas the reverse is usually true in practice. Actually, the dependence is quite weak, and its most serious influence is felt close to the plasma-vacuum boundary. This type of dependence on the density can here be modelled by the fact that, close to the vacuum boundary, the conductivities both decrease more rapidly than T^{k_0} and T^{k_1} respectively.

If the constants M, R and I_0 are selected as the definitive parameters (with independent dimensions), and conditions (2.6) are satisfied, the required functions can be written in the form

$$v(x, t) = \frac{I_0}{\sqrt{M}} \alpha(s) t^m, \qquad r(x, t) = \frac{I_0}{\sqrt{M}} \lambda(s) t^{m+1},$$

$$\rho(x, t) = \frac{M^2}{I_0^2} \delta(s) t^{-2(m+1)}, \qquad T(x, t) = \frac{I_0^2}{MR} f(s) t^{2m},$$

$$p(x, t) = M\beta(s) t^{-2}, \quad H_\varphi(x, t) = \sqrt{M}\, h(s) t^{-1},$$

$$E_z(x, t) = \frac{1}{c} I_0 \varphi(s) t^{m-1}, \qquad W(x, t) = I_0 \sqrt{M}\, \omega(s) t^{m-2}, \tag{2.7}$$

$$\sigma(x, t) = c^2 \frac{M}{I_0^2} \tilde{\sigma}(s) t^{-(1+2m)}, \qquad \varkappa(x, t) = RM\tilde{\varkappa}(s) t^{-1},$$

$$j_z(x, t) = c \frac{M}{I_0} \zeta(s) t^{-(m+2)}.$$

These expressions reveal how the various functions, in the self-modelling mode, depend on the problem parameters and time. For instance, the electrical resistance R_{pl} of the plasma per unit length of filament may be found from

$$R_{pl} = \left[2\pi \int_0^{r_*} \sigma r\, dr \right]^{-1} = \frac{R_0}{c^2} t^{-1}$$

(R_0 is a dimensionless quantity). This implies that, in the self-modelling mode, the resistance of the separating plasma decreases with time, but is independent of either the plasma mass M, the type of material (R), or the variation of the current I_0 (m).

The total energy contained in the volume occupied by the plasma is given by

$$e(t) = 2\pi \int_0^M \left(\varepsilon + 0.5v^2 + \frac{H_\varphi^2}{8\pi\rho} \right) dx = e_0 I_0^2 t^{2m} \tag{2.8}$$

(e_0 is a dimensionless constant).

When the self-modelling conditions (2.6) are satisfied, the equations of MRHD (2.1) reduce to a set of ordinary differential equations for the dimensionless functions α, β, δ, f, h, λ, ϕ, ω:

$$
\begin{gathered}
\alpha = (m+1)\lambda, \quad \lambda\beta' = -\frac{\zeta h}{\delta\lambda}, \quad \zeta = \tilde{\sigma}\varphi, \\
(\lambda h)' = 4\pi\frac{\zeta}{\delta}, \quad (\lambda\omega)' = -\frac{2m}{\gamma-1} f - 2(m+1)\frac{\beta}{\delta} + \frac{\zeta\varphi}{\delta}, \\
\omega = -\tilde{\varkappa}\lambda\delta f', \quad \beta = f\delta, \quad \tilde{\sigma} = \tilde{\sigma}_0 f^{k_0}\delta^{-q_0}, \quad \tilde{\varkappa} = \tilde{\varkappa}_0 f^{k_1}\delta^{-q_1}.
\end{gathered} \tag{2.9}
$$

The prime denotes differentiation with respect to the self-modelling variable s.

The dimensionless constants $\tilde{\sigma}_0$ and $\tilde{\kappa}_0$ are given in terms of the parameters M, I_0 and R, and σ_0 and κ_0 respectively, by

$$
\begin{aligned}
\tilde{\sigma}_0 &= \sigma_0 I_0^{2(k_0+q_0+1)}/M^{k_0+2q_0+1}R^{k_0}, \\
\tilde{\varkappa}_0 &= \varkappa_0 I_0^{2(k_1+q_1)}/M^{k_1+2q_1+1}R^{k_1+1}.
\end{aligned} \tag{2.10}
$$

The boundary conditions (2.2), (2.3) can be written in the self-modelling form as

$$\alpha(0) = 0, \quad h(0) = 0, \quad \omega(0) = 0, \tag{2.11}$$

$$\beta(1) = 0, \quad \lambda(1)h(1) = 2, \quad f(1) = 0. \tag{2.12}$$

3. We shall confine our future analysis of the self-modelling solutions to the case of fixed current ($m = 0$).

Here, under the extra assumption that $k_0 = q_0 = 0$, the solution may be obtained in analytical form. In this case the electrical and thermal conductivities are

$$\sigma = \sigma_0 t^{-1}, \quad \varkappa = \varkappa_0 T^{k_1}\rho^{-q_1}t^{-(1+2q_1)}. \tag{3.1}$$

The time-dependences of σ and κ in (3.1) are extremely artificial from the physical stand-point. But computations of the set (2.9)–(2.12) show that the main qualitative features of the solution obtained in this elementary particular case are retained in more general circumstances, given reasonable values of the constants k_0, q_0, n_0, k_1, q_1, n_1.

Obviously, when the discharge current is fixed in the self-modelling solution, the total plasma energy, and hence any quantity with the dimensions of energy, must be time-independent by virtue of (2.8). The constant energy condition is also satisfied in the self-modelling solutions of the problem on a strong explosion in the atmosphere obtained in [8, 9], where the energy released at the initial instant remains fixed throughout the future process. In our problem of an electric discharge in a plasma, constant energy is achieved, not by the system having conservative properties, but by the balance between the electromagnetic energy entering the system, and the energy dissipated in work against the forces of the magnetic field, plus the energy leaving the system in the form of heat flux. Non-trivial self-modelling solutions of this kind are clearly impossible in ordinary gas dynamics. Their existence depends on the presence of supplementary external sources of energy such as Joule heat.

Integration of (2.9) subject to the condition (3.1), $m = 0$, and say $q_1 > -1$, leads to the following expressions for the dimensionless functions of velocity α, pressure β, magnetic field strength h, temperature f, density δ and heat flux ω in terms of the dimensionless radius λ:

$$\alpha = \lambda, \quad \beta = \frac{1}{\pi\lambda_*^4}(\lambda_*^2 - \lambda^2), \quad h = \frac{2}{\lambda_*^2}\lambda, \quad \omega = \frac{\lambda}{2\pi\lambda_*^4}\left(\frac{1}{\pi\tilde{\sigma}_0} - 2\lambda_*^2 + \lambda^2\right), \tag{3.2}$$
$$f = \{A[(B-1)\lambda_*^2 + \lambda^2](\lambda_*^2 - \lambda^2)^{q_1+1}\}^{1/(k_1+q_1+1)}, \qquad \delta = \beta f^{-1},$$

where

$$A = \frac{k_1 + q_1 + 1}{4\tilde{\varkappa}_0 (q_1 + 2)\pi^{q_1+1}\lambda_*^{4(q_1+1)}}, \qquad B = \frac{q_1+2}{q_1+1}\left(\frac{1}{\pi\tilde{\sigma}_0\lambda_*^2} - 1\right).$$

The relationship $s = \int_0^\lambda \beta f^{-1}\lambda\, d\lambda$ gives the connection between the dimensionless radius and the self-modelling variable. The dimensionless radius λ_* of the plasma-vacuum boundary is found from the condition for the plasma mass to be constant:

$$1 = \int_0^{\lambda_*} \frac{\beta}{f}\lambda\, d\lambda = \frac{1}{\pi\lambda_*^4 A^{1/(k_1+q_1+1)}}\int_0^{\lambda_*}\left\{\frac{(\lambda_*^2 - \lambda^2)^{k_1}}{(B-1)\lambda_*^2 + \lambda^2}\right\}^{1/(k_1+q_1+1)}\lambda\, d\lambda. \tag{3.3}$$

The electric field strength and current density are constant in this solution:

$$\varphi = \frac{1}{\pi\tilde{\sigma}_0\lambda_*^2}, \qquad \zeta = \tilde{\sigma}_0\varphi = \frac{1}{\pi\lambda_*^2}. \tag{3.4}$$

It follows from (3.2) that the pressure is a monotonically decreasing function of the radius, while h increases with λ. The temperature f is not necessarily a monotonic function of λ. The position λ_{max} of its maximum is given by

$$\lambda_{\max}^2 = 2\lambda_*^2 - \frac{1}{\pi\tilde{\sigma}_0}, \tag{3.5}$$

while the maximum of f is

$$f_{\max} = \left[\frac{k_1 + q_1 + 1}{4\tilde{\varkappa}_0 (q_1 + 1)(q_1 + 2)\pi^{q_1+1}\lambda_*^{2q_1}}\left(\frac{1}{\pi\tilde{\sigma}_0\lambda_*^2} - 1\right)^{q_1+2}\right]^{1/(k_1+q_1+1)}$$

For the temperature maximum to occur inside the range $0 < \lambda_{\max} < \lambda_*$, we must have

$$\frac{1}{2\pi\tilde{\sigma}_0} < \lambda_*^2 < \frac{1}{\pi\tilde{\sigma}_0}. \tag{3.6}$$

It follows from the expression for $f(\lambda)$ in (3.2) that the solution is meaningful ($f(\lambda) \geqslant 0$) throughout the range $0 \leqslant \lambda \leqslant \lambda_*$ only when $B \geqslant 1$ or

$$\lambda_*^2 \leqslant \frac{q_1 + 2}{2q_1 + 3}\frac{1}{\pi\tilde{\sigma}_0}. \tag{3.7}$$

Comparing (3.6) and (3.7), we can now conclude that the temperature is not in fact monotonic in the solution; its maximum lies inside the interval $(0, \lambda_*)$ when

$$\frac{1}{2\pi\tilde{\sigma}_0} < \lambda_*^2 \leqslant \frac{q_1 + 2}{2q_1 + 3}\frac{1}{\pi\tilde{\sigma}_0},$$

or what amounts to the same thing,

$$2 < \mathrm{Re}_m^* \leqslant 4\frac{q_1 + 2}{2q_1 + 3}, \tag{3.8}$$

where $\mathrm{Re}_{m*} = 4\pi\tilde{\sigma}_0\lambda^2$ is the magnetic Reynold's number, evaluated from the value of the velocity of the plasma-vacuum boundary and its distance from the centre. When $\mathrm{Re}_m^* \leqslant 2$, the temperature maximum is always reached on the axis, and $f(\lambda)$ is a monotonically decreasing function. When $\mathrm{Re}_m^* > 4(q_1 + 2)/(2q_1 - 3)$, the self-modelling solution becomes meaningless.

It can be seen from the above inequalities that the characteristic magnetic Reynold's number must be high if the temperature relation is to have non-monotonic properties (existence of a T layer) in the solution of the heavy current discharge problem. This conclusion agrees with the conditions obtained in [4, 6] for the existence of a T layer.

Consider the dependence of the self-modelling solution (3.2) on the thermal conductivity. We shall first take the simplest case, when κ is temperature- and density-independent ($k_1 = q_1 = 0$, $\kappa = \kappa_0 t^{-1}$). We can find λ_* explicitly from (3.3):

$$\lambda_*^2 = \frac{2}{3\pi\tilde{\sigma}_0} \frac{\exp(^1/_4\,\tilde{\varkappa}_0) - 1}{\exp(^1/_4\,\tilde{\varkappa}_0) - ^2/_3}.$$

From (3.7), a self-modelling solution only exists in this case when $\tilde{\varkappa}_0 > \tilde{\varkappa}_{01} = 0$.

Condition (3.8) for non-monotonic temperature relations can be rewritten as

$$\tilde{\varkappa}_{01} < \tilde{\varkappa}_0 < \tilde{\varkappa}_{02}, \qquad \tilde{\varkappa}_{01} = 0, \qquad \varkappa_{02} = \frac{1}{4\ln 2}. \tag{3.9}$$

When $\tilde{\varkappa}_0 \geqslant \tilde{\varkappa}_{02}$, the temperature maximum occurs at the centre.

Now consider the simplest case of non-linear thermal conductivity: $k_1 = 1$, $q_1 = 0$, $\tilde{\varkappa} = \tilde{\varkappa}_0 T t^{-1}$. Working similar to the above again leads to the inequality (3.9) with $\tilde{\varkappa}_{01} = 4/\pi$ and $\tilde{\varkappa}_{02} = 4/\pi(1 - 2/\pi)^2$; when $\tilde{\varkappa}_0 < \tilde{\varkappa}_{01}$, the self-modelling solution becomes meaningless, since a region with negative temperature appears in it. When $\tilde{\varkappa}_0 > \tilde{\varkappa}_{01}$, the dependence of the self-modelling solution on $\tilde{\varkappa}_{01}$ is as shown by the curves of Fig. 2 ($\tilde{\sigma}_0 = 0.02$, $k_1 = 1$, $q_1 = 0$). As $\varkappa_0$ increases, the temperature maximum falls and moves closer to the axis. As $\tilde{\varkappa}_0 \to \infty$, $f(0) = 1/3\pi$.

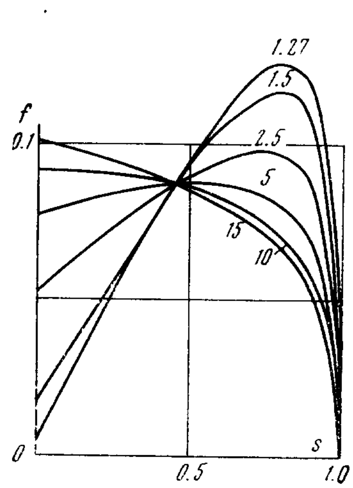

FIG. 2.

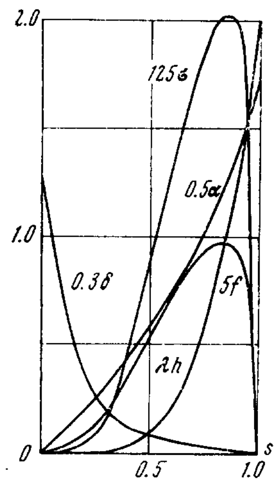

FIG. 3.

To sum up, it has been found in two simple particular cases that the range in which a self-modelling solution is possible has a lower limit $\tilde{\varkappa}_{01}$. Analysis of the solution (3.2) shows that, as the index k_1 increases, i.e., as the thermal conductivity becomes more non-linear, the nature of the solution remains unchanged, while the value of $\varkappa_{01}$ increases.

4. Under more general assumptions than conditions (3.1), derivation of the self-modelling solution of the problem of plasma separation in a vacuum amounts to numerical solution of the ordinary differential equations (2.9) subject to the conditions (2.11) and (2.12). Computations show that the main qualitative features of the self-modelling solution remain the same as in the particular cases examined analytically above. As an example, Fig. 3 shows typical distributions of the required dimensionless functions over the self-modelling variable, obtained by computations for the problem with the parameter values $k_0 = 3/2$, $q_0 = 0$, $k_1 = 1$, $q_1 = 0$, $\kappa_0 = 1.5$, $\sigma_0 = 0.2$.

Here, the plasma conductivity $\tilde{\sigma}$, and the current density ξ, are no longer constant; the maximum of $\tilde{\sigma}$ coincides with the maximum of the temperature f.

By considering the dependence of the solution on the parameter κ_0, it can be shown that, in this case also, there are two characteristic values $\tilde{\kappa}_{01}$ and $\tilde{\kappa}_{02}$ of the thermal conductivity. The self-modelling solution only exists when $\tilde{\kappa}_0 > \tilde{\kappa}_{01}$; the temperature profile is non-monotonic with respect to s in the range $\tilde{\kappa}_{01} < \tilde{\kappa}_0 < \tilde{\kappa}_{02}$, and the maximum occurs at the centre when $\tilde{\kappa}_0 \geqslant \tilde{\kappa}_{02}$.

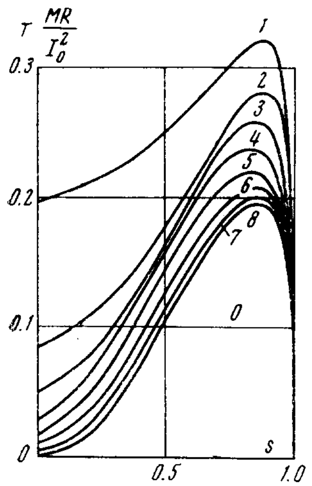

FIG. 4.

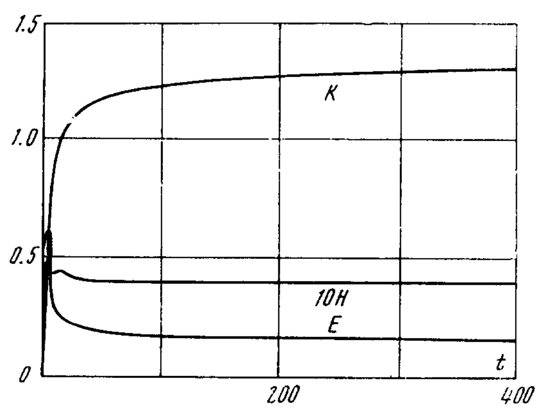

FIG. 5.

In short, even when the self-modelling conditions (2.6), obtained from ordinary dimensional analysis, are satisfied, the self-modelling solution does not exist for all values of the thermal conductivity $\tilde{\kappa}_0$, even though the latter does not appear formally in the conditions (2.6).

5. The self-modelling solutions plotted here were obtained by numerical solution of the full set of equations (2.1). For this purpose, the set of differential

equations was approximated by a homogeneous, completely conservative difference scheme, which was solved by the method of successive pivoted condensations [10–12]. The boundary states were realized in accordance with (2.2) and (2.3) The initial conditions were specified in the form of arbitrary functions of the x coordinate, not the same as the self-modelling profiles. This numerical solution of the problem, with the same parameter values as in Fig. 3, is shown in Fig. 4 for $\tilde{\kappa}_{01} < \tilde{\kappa}_0 < \tilde{\kappa}_{02}$. The temperature profiles are drawn at successive instants, with intervals between them such that the amount of electromagnetic energy entering the plasma during one interval is always the same; the amount entering in the initial interval is equal to two. Here, $(MR/I_0^2)\,T(x,\,0) = 0.1$, $m = 0$.

In time, the solution moves into the self-modelling mode.

Figure 5 shows the time-variations of the different types of plasma energy in the dimensionless form:
the internal energy

$$E = \frac{1}{I_0^2}\int_0^M \varepsilon\, dx,$$

the kinetic energy

$$K = \frac{1}{I_0^2}\int_0^M 0.5\, v^2\, dx$$

and the magnetic energy

$$H = \frac{1}{I_0^2}\int_0^M \frac{H_\varphi^2}{8\pi\rho}\, dx.$$

As t increases, the values of these quantities, and also the total energy, tend to their values in the self-modelling solution.

The set (2.1) was also solved numerically in the 'almost self-modelling' region, i.e., when conditions (2.6) are satisfied but the thermal conductivity has low values: $\tilde{\kappa}_0 < \tilde{\kappa}_{01}$. The solutions here were not in fact of the self-modelling type, and the behaviour of the flow parameters ceased to remain within the framework of relationships (2.7). A solution which is not self-modelling is of an essentially non-stationary type; the appearance and development of a high-temperature T layer are observed, together with various other phenomena that usually accompany this, such as the formation of a shock wave, travelling along the axis, over-all braking of the gas, and pinching of the plasma filament etc. [3, 4].

On combining the facts obtained from an analysis of the self-modelling solutions and the results from numerical computations in the 'almost self-modelling' region, various conclusions can be drawn regarding the influence of the thermal conductivity on the processes occurring in a heavy-current discharge plasma. When the thermal conductivity is low ($\tilde{\kappa}_0 \leqslant \tilde{\kappa}_{01}$), a high-temperature T-layer appears and develops in the plasma. The solution here is not at all of the self-modelling type. For instance, the gas temperature increases in the T-layer, whereas it falls in the central region.

When $\tilde{\kappa}_{01} < \tilde{\kappa}_0 < \tilde{\kappa}_{02}$, the influence of the thermal conductivity is already strong enough to produce a particular kind of T-layer stabilization as a result of the out-flow of heat: the temperature of the entire mass of gas varies with time according to the same power law.

A high thermal conductivity destroys the T layer, the temperature is found to have only monotonic properties in the solution, and its maximum is located on the axis.

6. A special feature of the analytic solution obtained in Section 3 may be mentioned. The temperature maximum appearing in this solution (in both the self-modelling and the 'almost self-modelling' regions) cannot be called a T-layer in the full meaning of the word, since here $\sigma_0 = \sigma_0 t^{-1}$ is temperature-independent, while a condition for the appearance of such a layer is the existence of considerable non-linearity: $d \ln \sigma / dT > 0$. The classical scanning is also absent in this case, since the plasma conductivity becomes quite small as t increases, while the current density becomes constant over the radius.

Nevertheless, the temperature has a pronounced maximum. The existence of this maximum is explained by the dependence of the Joule heat per unit mass on the density: $Q = \zeta^2 / \tilde{\sigma}_0 \delta$.

In the present problem, due to the strong separation, the density δ falls on approaching the plasma-vacuum boundary, and the Joule heat correspondingly increases. This behaviour of Q leads to the appearance of a temperature maximum, though the position of it is not the same as that of the Joule heat maximum, due to the heat conduction processes.

There is thus a certain limiting point at which the T-layer effect degenerates; this point corresponds to the absence of feedback between the gas-dynamical and electromagnetic processes (the electric conductivity is independent of the thermodynamic state of the medium).

If, however, the conductivity is a function of the temperature, this type of inhomogeneous state can become fundamental for development of the *T*-layer. Thus, in addition to the familiar skin effect and overheating instability, a further possible mechanism of *T*-layer initiation may be seen.

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